PIE Tech

POLLACHI INSTITUTE OF ENGINEERING AND TECHNOLOGY

(Approved by AICTE and Affiliated to Anna University)

sky is the limit

Department of Electronics and Communication Engineering

Regulation 2021

II Year – III Semester

EC3351 - Control Systems

UNIT - I

Systems and Representation.

Syllabus :-

Basic elements in control System - open Loop and closed Loop Systems - Electrical Analogy of Mechanical and Thermal Systems - Transfer Function - Ac and DC Servomotors - Block reduction Techniques - Signal flow graph.

System :-

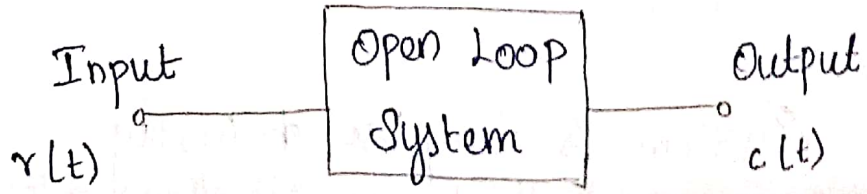
When a number of components (or) elements are connected in sequence to perform a specific task is called as a System.

Control System :-

The output quantity is controlled by varying the Input quantity is called as control System.

2m

Open Loop System :-



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This Output is not controlled (does not correct automatically) is called open loop System.

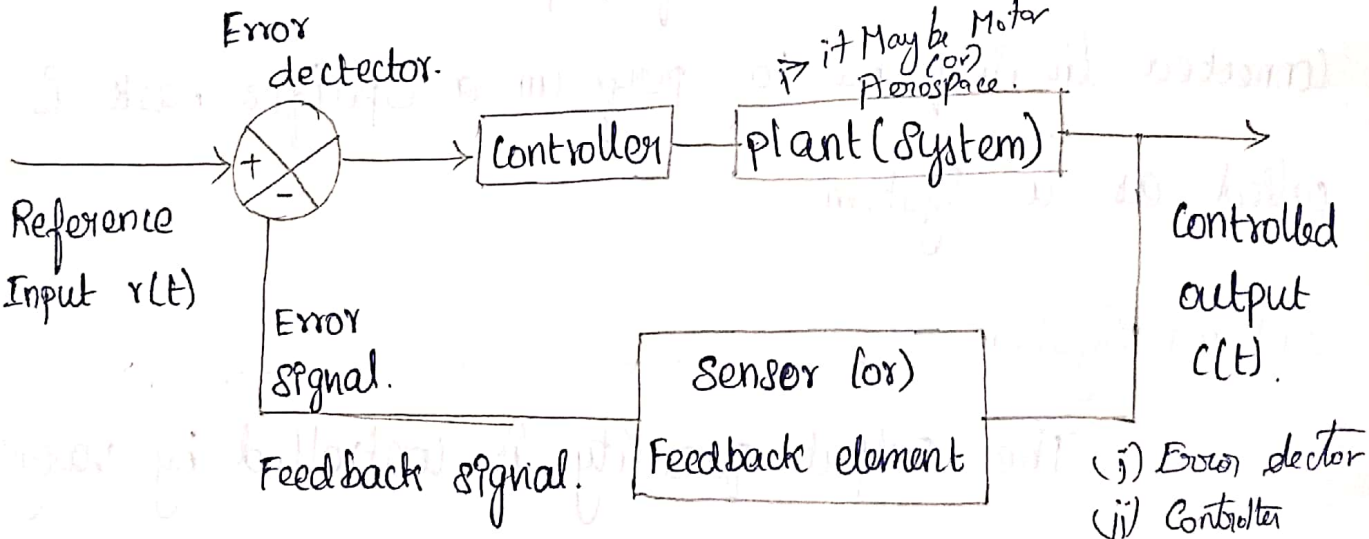
Advantages:-

* Simple in construction.
* Easy Maintenance.

Disadvantage:-

* Inaccurate results.
* cannot sense the Internal disturbance.

Closed Loop System (Basic elements in control System)
(Maintain the desired output value).



Advantages:-

* Systems are accurate.
* Closed loop system acts very Fasts.

Disadvantages:-

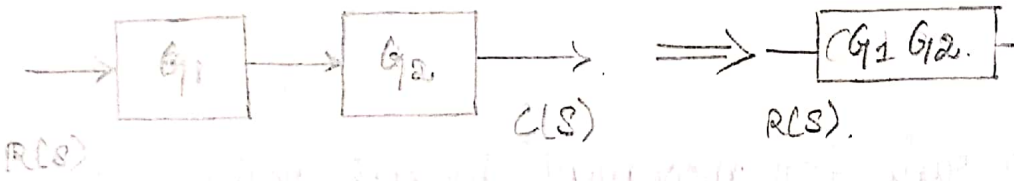
* System is complex & costly.
* Because of feedback, the Overall gain of the system is Reduced.

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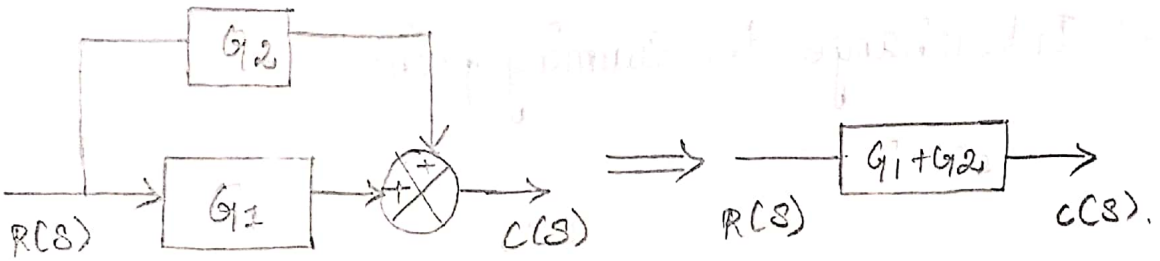
Block diagram Reduction.

1. Block Diagram Reduction
2. Signal Flow graph
3. Mechanical Translational s/m.
4. Mechanical / Rotational s/m.
5. E.A. of M-T-S
6. E.A of M.R.S.
7. AC & DC servometer

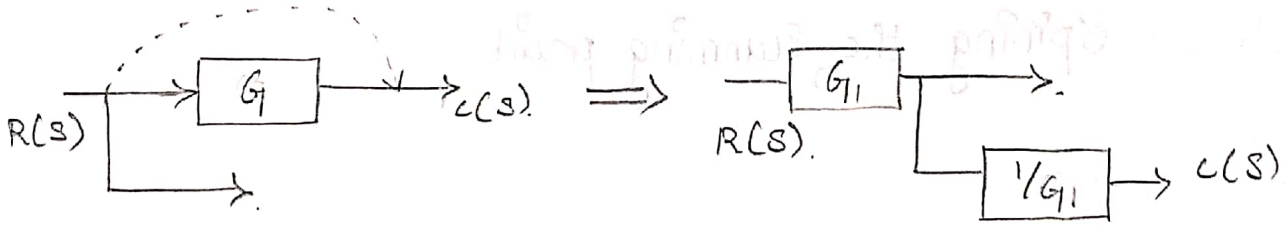
Rule 1:- Combine the block in Series.



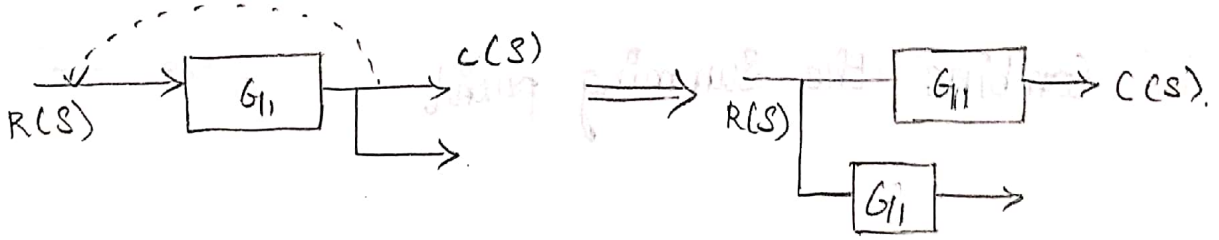
Rule 2:- Combine the block in parallel.



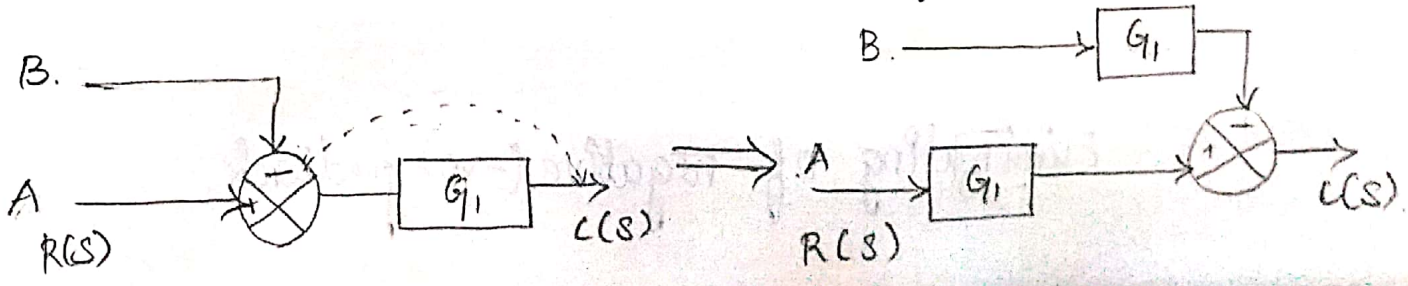
Rule 3:- Move the branch point (after) the block.



Rule 4:- Move the branch point before the block.

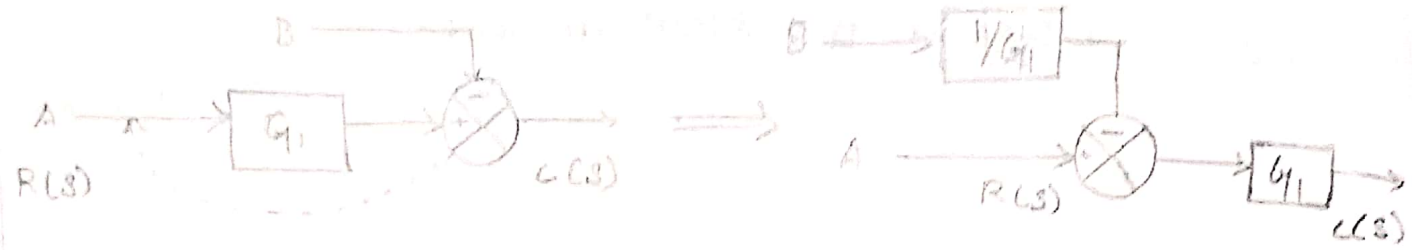


Rule 5:- Move the Summary point after the block.



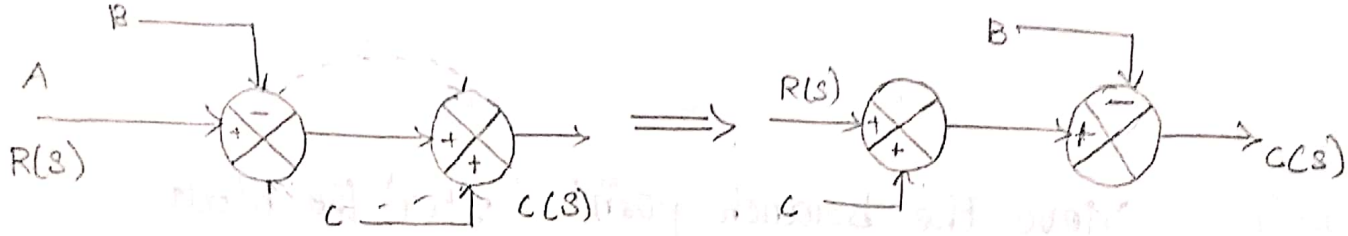
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Rule 6 :- Move the Summary point the block.

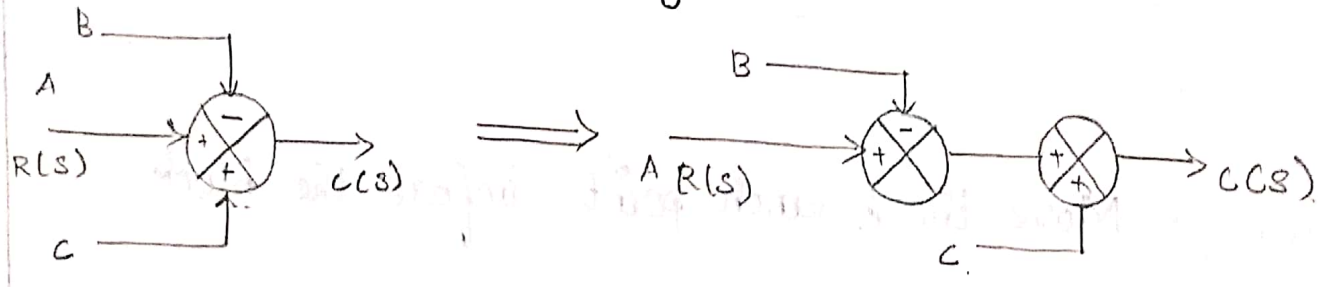


(There is no rule for movement branch point after Summing point).

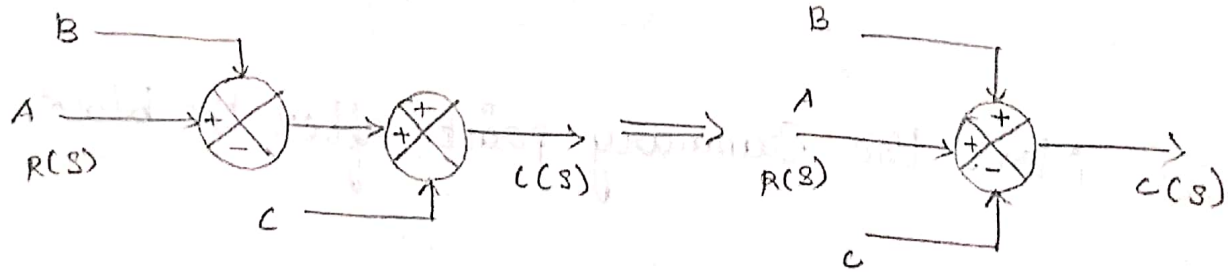
Rule 7 :- Interchange the Summing point.



Rule 8 :- Spining the Summing point.



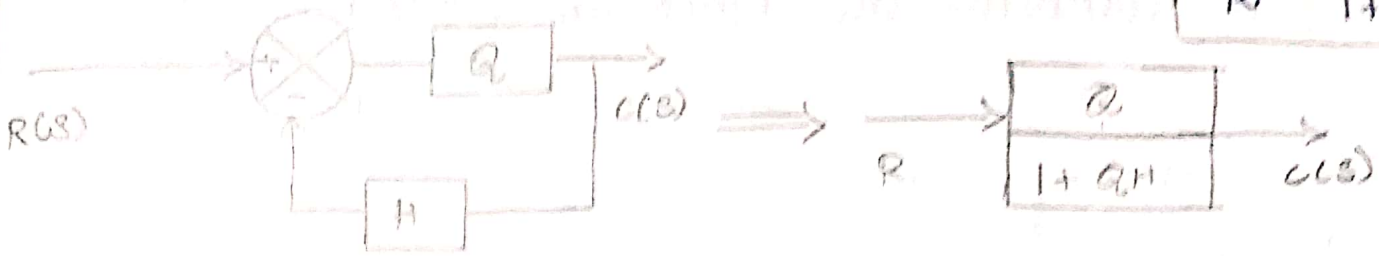
Rule 9 :- Combine the Summing point.



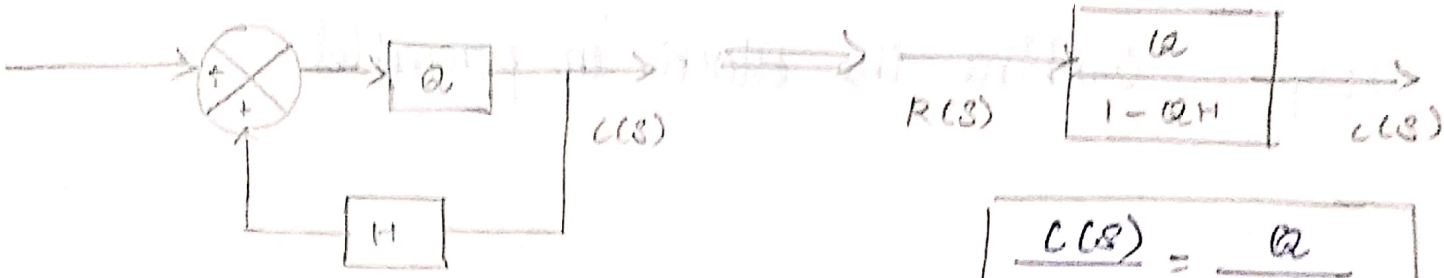
Rule 10 :- Eliminating of negative (-ve) feedback.

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$\frac{C}{R} = \frac{G}{1+GH}$

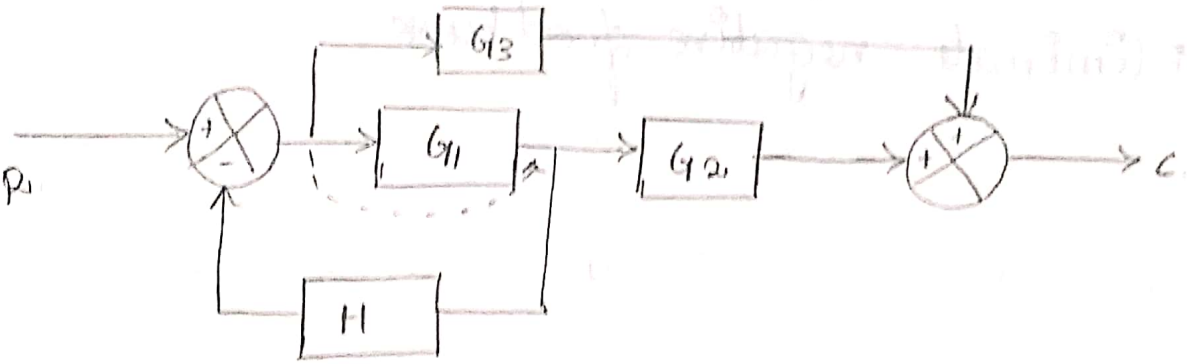


Rule II :- Elimination of positive feedback.

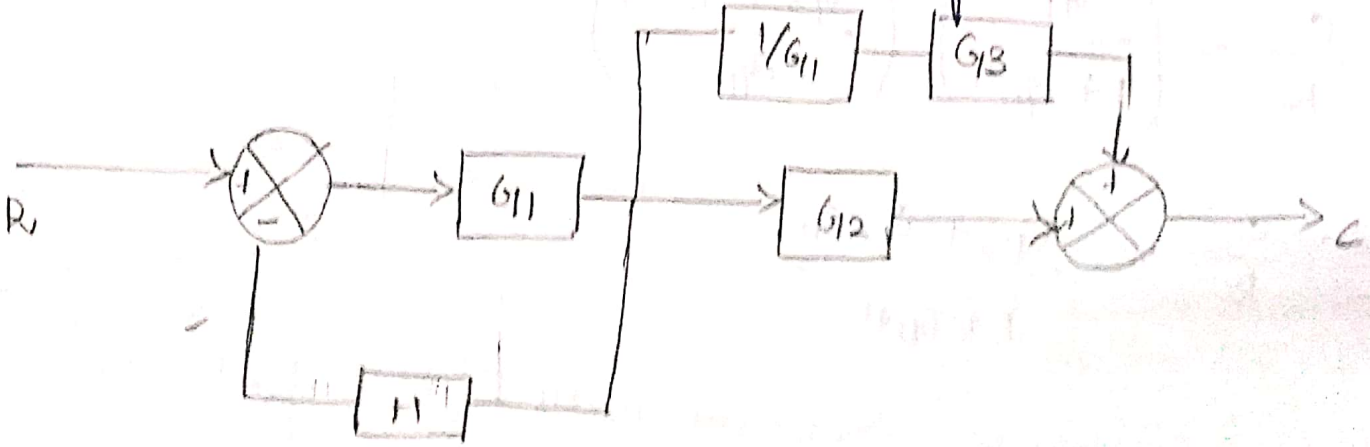


$$\frac{C(s)}{R(s)} = \frac{G}{1-GH}$$

1. Reduce the Block diagram to find C/R :

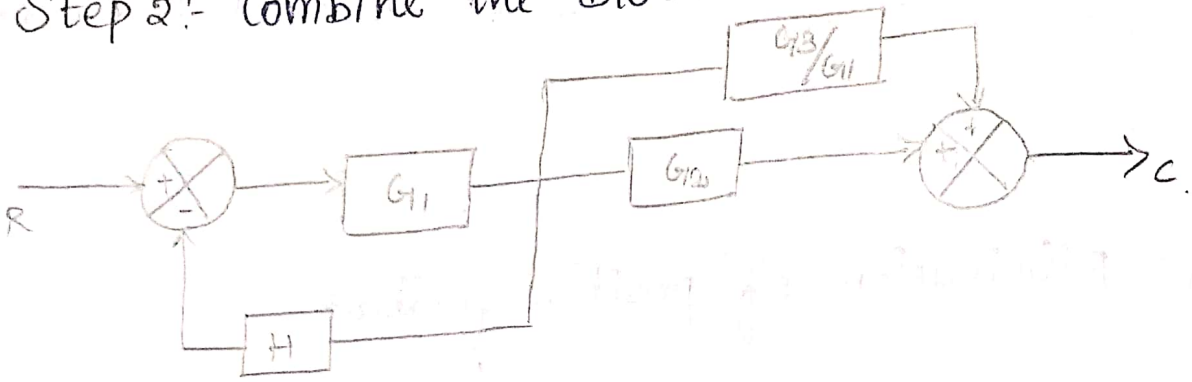


Step 1 :- Move the Branch point after the Block

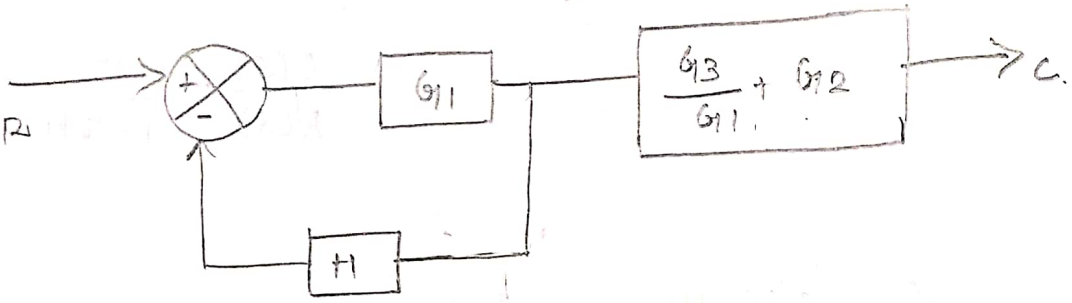


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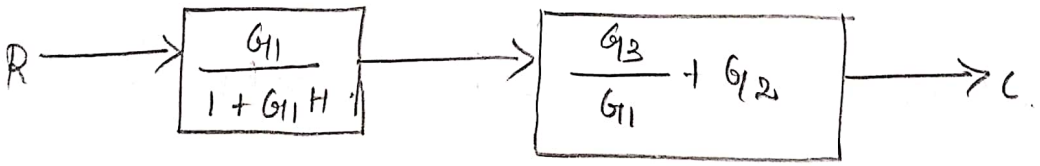
Step 2:- Combine the Block in Series.



Step 3:- Combine the Block in parallel.



Step 4:- Eliminate negative feedback.

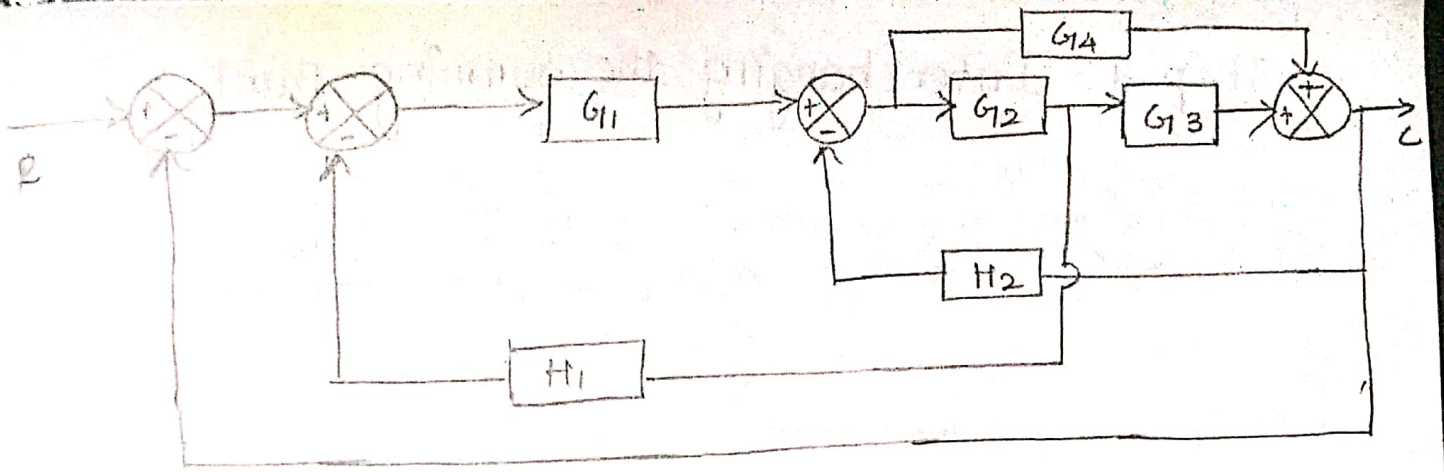


$$\frac{C}{R} = \left(\frac{G_1}{1 + G_1 H}\right)\left(\frac{G_3 + G_2 G_1}{G_1}\right)$$

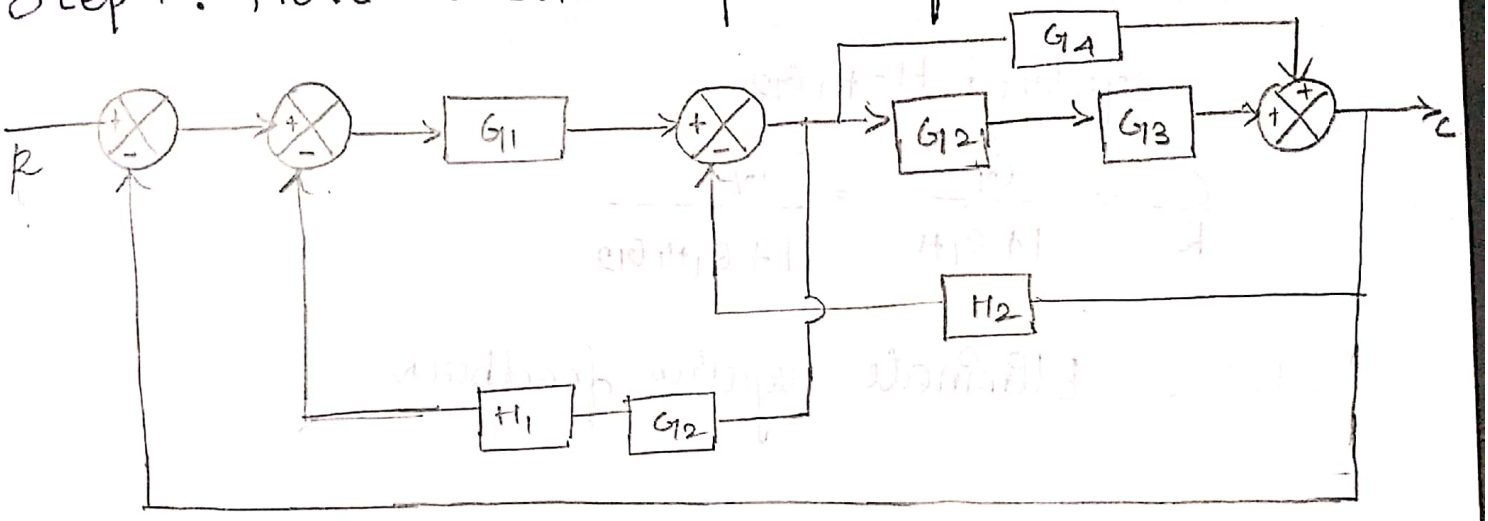
$$\frac{C}{R} = \frac{G_3 + G_2 G_1}{1 + G_1 H}$$

2. Using Block Diagram Reduction to find C/R:-

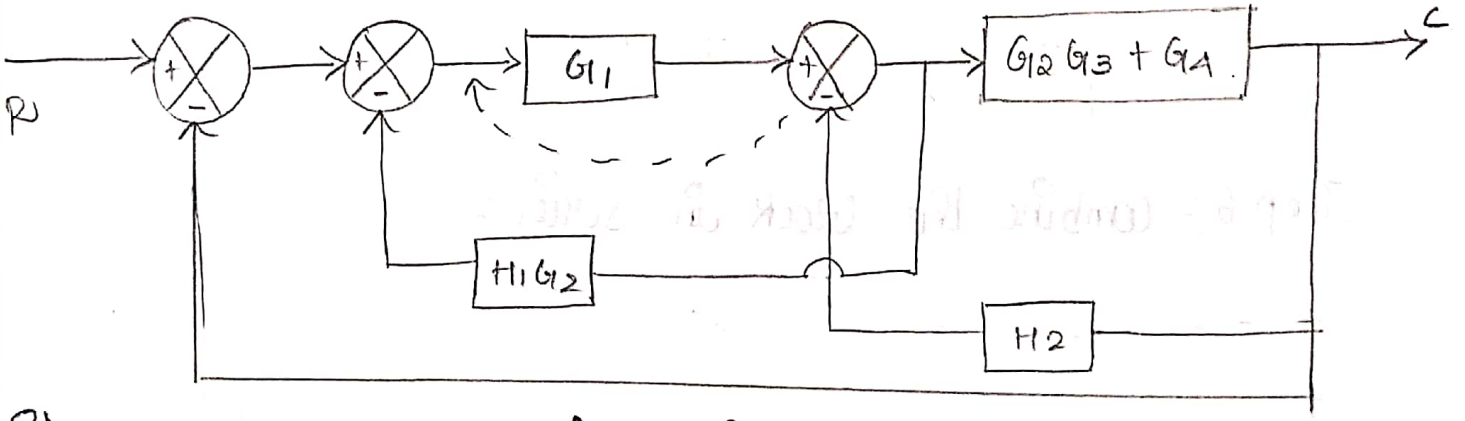
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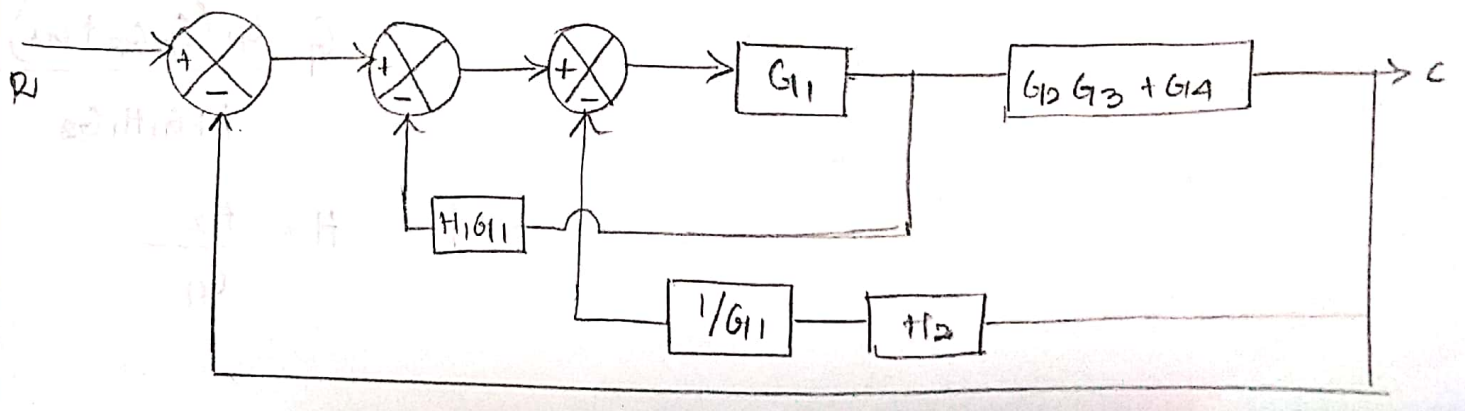
Step 1 :- Move the Branch point before the block.



Step 2 :- combine the block in Series and parallel.

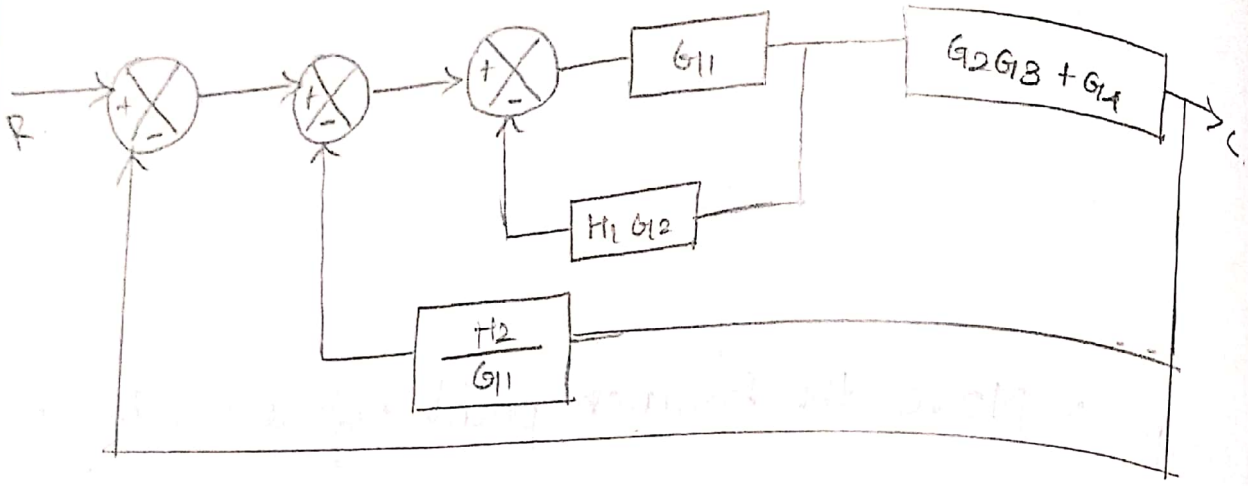


Step 3 :- Move the summing point below the block.



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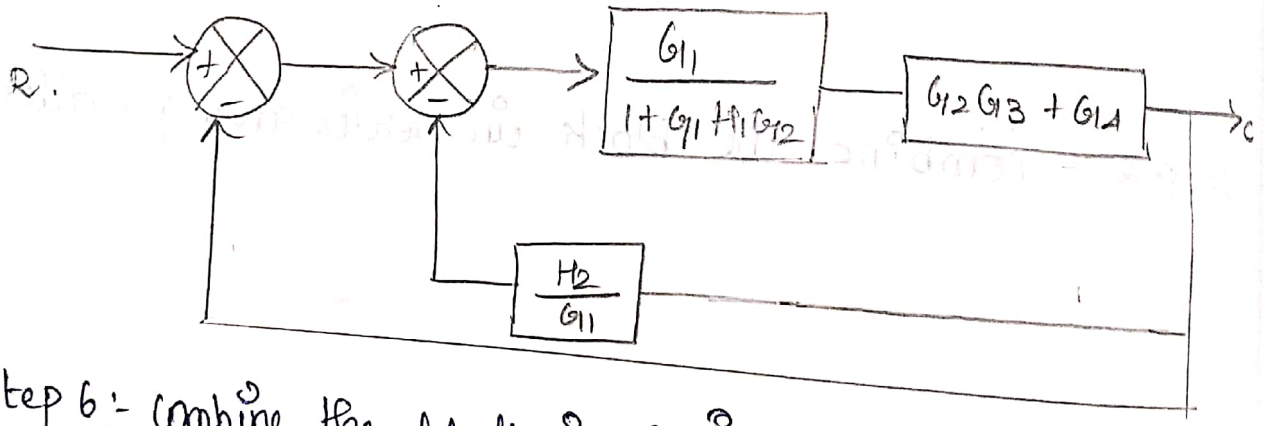
Step 4:- Interchanging the Summing point.



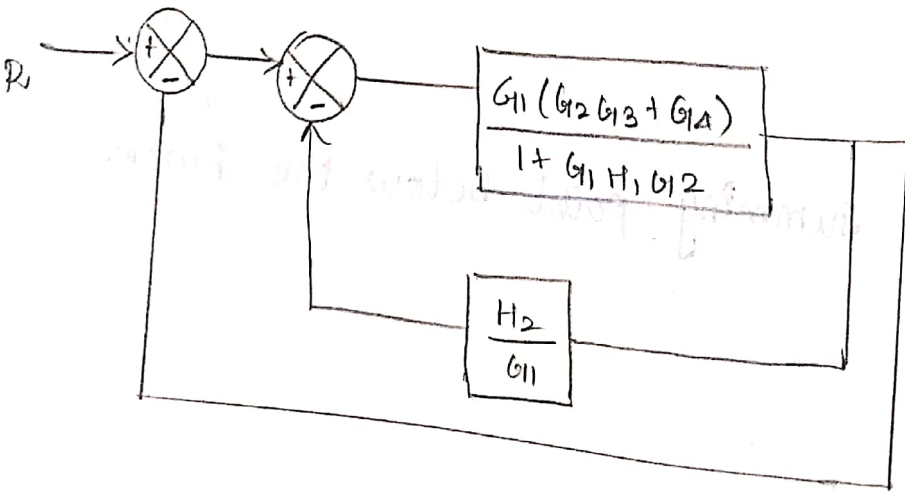
$G = G_{11}$; $H = H_1 G_{12}$.

$$\frac{C}{R} = \frac{G}{1+GH} = \frac{G_{11}}{1+G_{11}H_1G_{12}}.$$

Step 5:- Eliminate negative feedback.



Step 6:- Combine the block in Series :-



$$G = \frac{G_{11}(G_2G_3 + G_4)}{1 + G_1H_1G_2}$$

$$H = \frac{H_2}{G_1}$$

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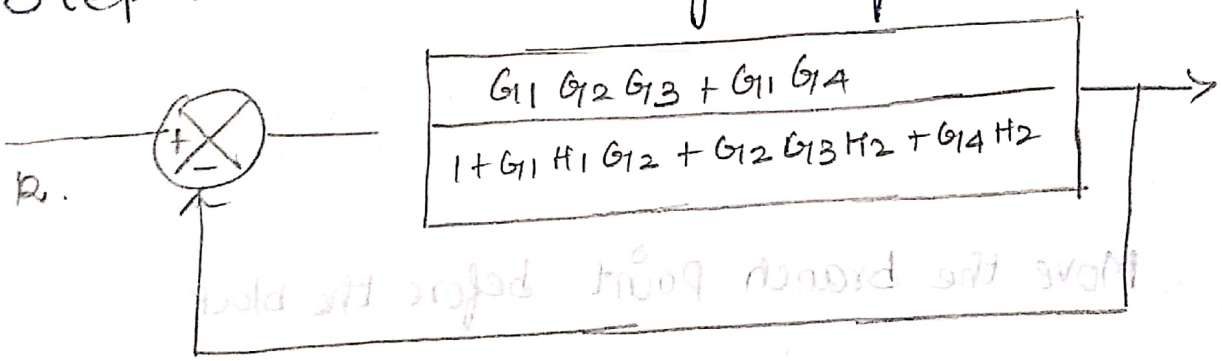
$$\frac{G}{1+GH} = \frac{G_1 (G_2 G_3 + G_4)}{1 + G_1 H_1 G_2}$$

$$\frac{1 + \dfrac{G_1 (G_2 G_3 + G_4)}{1 + G_1 H_1 G_2} \cdot \dfrac{H_2}{G_1}}{}$$

$$= \frac{\dfrac{G_1 G_2 G_3 + G_1 G_4}{1 + G_1 H_1 G_2}}{\dfrac{1 + G_1 H_1 G_2 + G_2 G_3 H_2 + G_4 H_2}{1 + G_1 H_1 G_2}}$$

$$= \frac{G_1 G_2 G_3 + G_1 G_4}{1 + G_1 H_1 G_2 + G_2 G_3 H_2 + G_4 H_2}$$

Step 7 :- Eliminate negative feedback.



$$G = \frac{G_1 G_2 G_3 + G_1 G_4}{1 + G_1 H_1 G_2 + G_2 G_3 H_2 + G_4 H_2} \quad ; H = 1$$

$$\frac{G}{1+GH} = \frac{G_1 G_2 G_3 + G_1 G_4}{1 + G_1 H_1 G_2 + G_2 G_3 H_2 + G_4 H_2}$$

$$\frac{}{1 + \dfrac{G_1 G_2 G_3 + G_1 G_4}{1 + G_1 H_1 G_2 + G_2 G_3 H_2 + G_4 H_2} \cdot 1}$$

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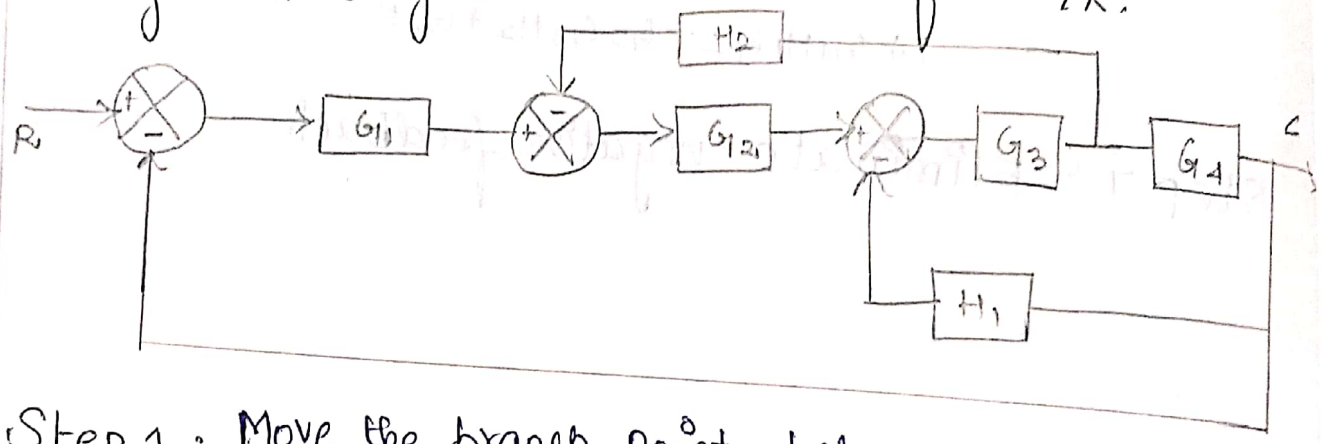
$$\frac{C}{R} = \frac{G_1 G_2 G_3 + G_1 G_4}{1 + G_1 H_1 G_2 + G_2 G_3 H_2 + G_4 H_2}$$

$$\frac{C}{R} = \frac{G_1 G_2 G_3 + G_1 G_4}{1 + G_1 H_1 G_2 + G_2 G_3 H_2 + G_4 H_2 + G_1 G_2 G_3 + G_1 G_4}$$

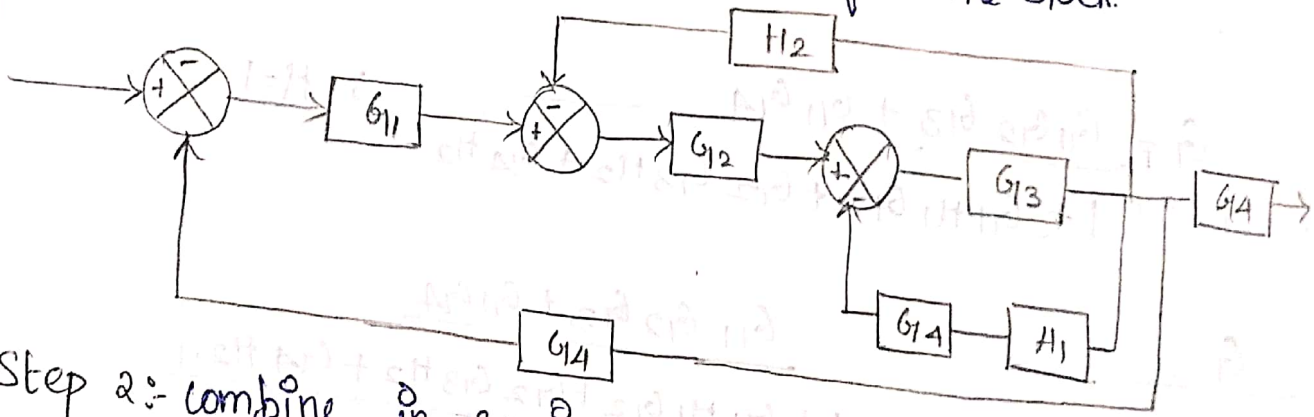
$$\frac{C}{R} = \frac{G_1 G_2 G_3 + G_1 G_4}{1 + G_1 H_1 G_2 + G_2 G_3 H_2 + G_4 H_2}$$

$$\frac{C}{R} = \frac{G_1 G_2 G_3 + G_1 G_4}{1 + G_1 H_1 G_2 + G_2 G_3 H_2 + G_4 H_2 + G_1 G_2 G_3 + G_1 G_4}$$

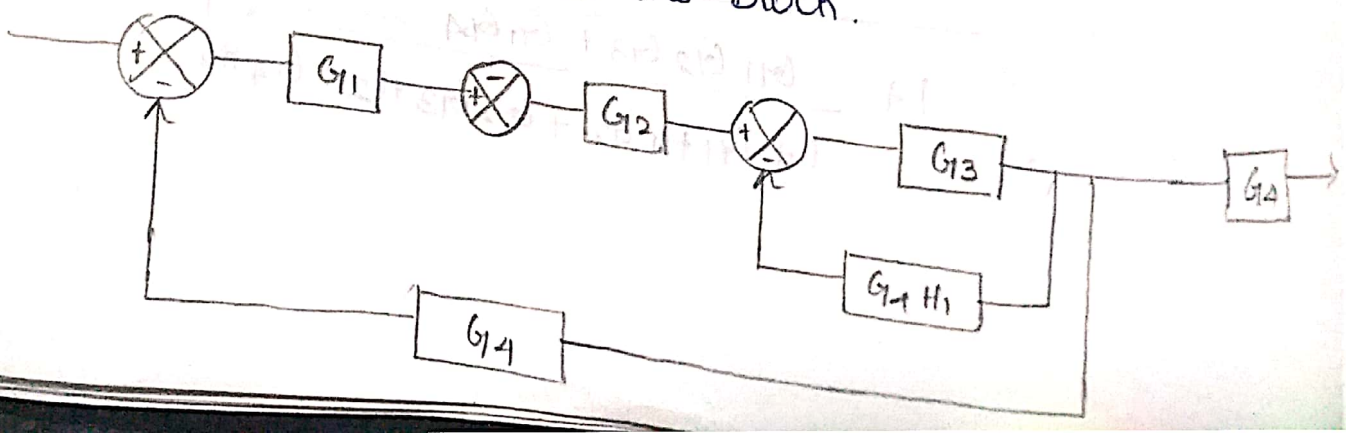
Using block diagram reduction to find C/R.



Step 1 :- Move the branch point before the block.

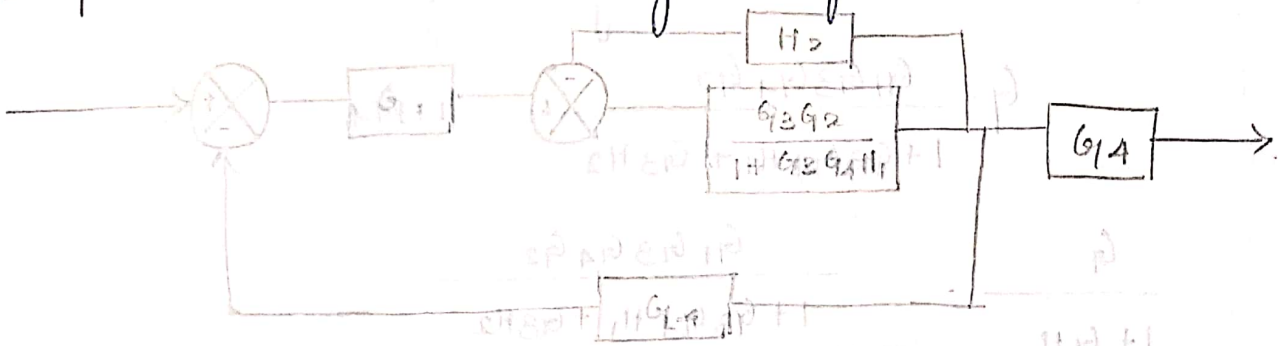


Step 2 :- combine in series the block.



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Step 3:- Eliminate the negative feedback.

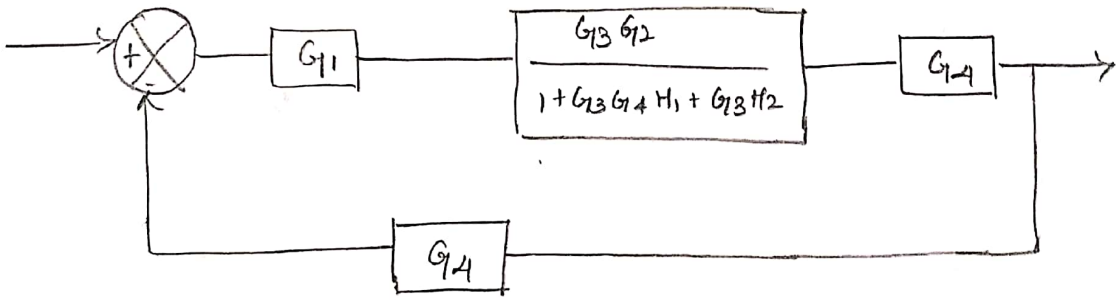


Eliminate the negative feedback.

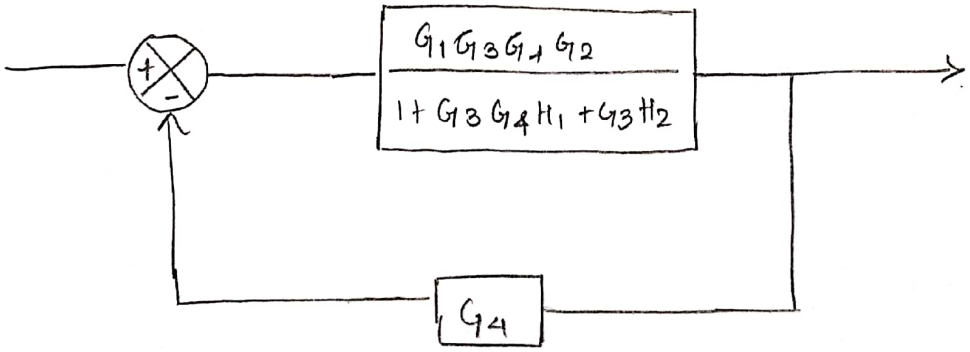
$$G = \frac{G_3 G_2}{1 + G_3 G_4 H_1} \qquad H = H_2.$$

$$\frac{G}{1 + GH} = \frac{\frac{G_3 G_2}{1 + G_3 G_4 H_1}}{1 + \left(\frac{G_3}{1 + G_3 G_4 H_1}\right) H_2}$$

$$= \frac{G_3 G_2}{1 + G_3 G_4 H_1 + G_3 H_2}$$

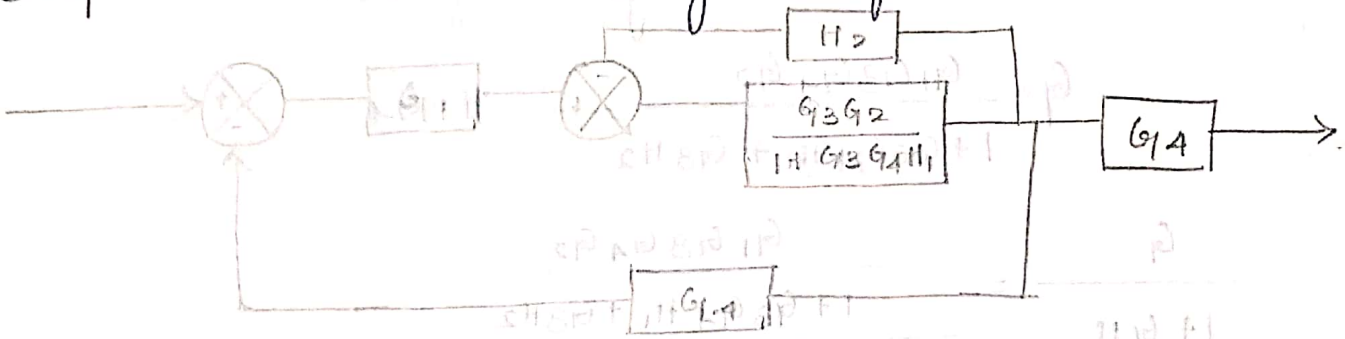


Step 5:- Combine the block in Series.



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Step 3:- Eliminate the negative feedback.

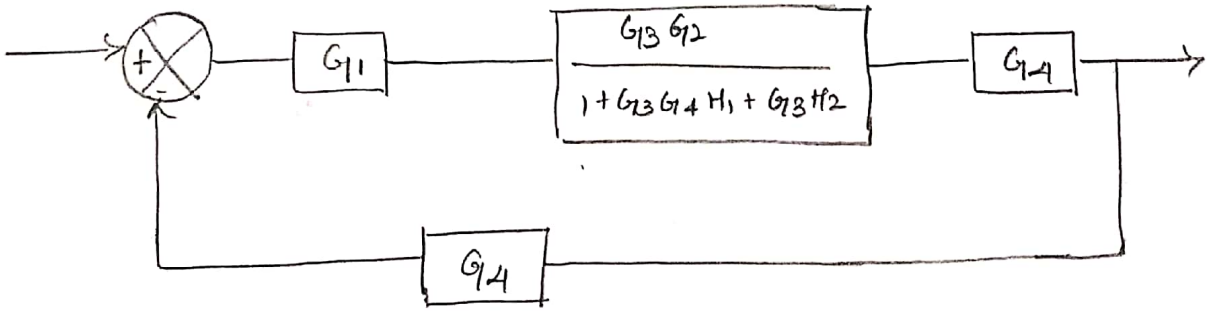


Eliminate the negative feedback.

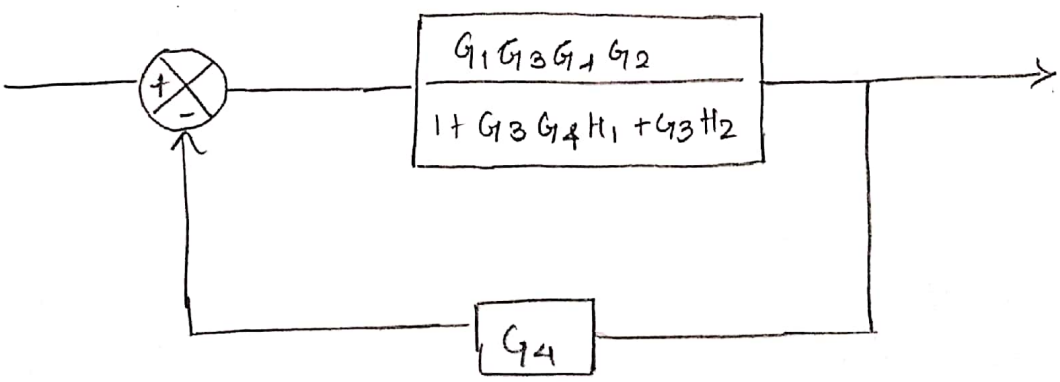
$$G_1 = \frac{G_3 G_2}{1 + G_3 G_4 H_1} \qquad H = H_2.$$

$$\frac{G_1}{1 + G_1 H} = \frac{\frac{G_3 G_2}{1 + G_3 G_4 H_1}}{1 + \left(\frac{G_3}{1 + G_3 G_4 H_1}\right) H_2}$$

$$= \frac{G_3 G_2}{1 + G_3 G_4 H_1 + G_3 H_2}$$



Step 5:- Combine the block in Series.



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Step 6 :- Eliminate the negative feedback.

$$G = \frac{G_1 G_3 G_4 G_2}{1 + G_3 G_4 H_1 + G_3 H_2} \qquad H = G_4.$$

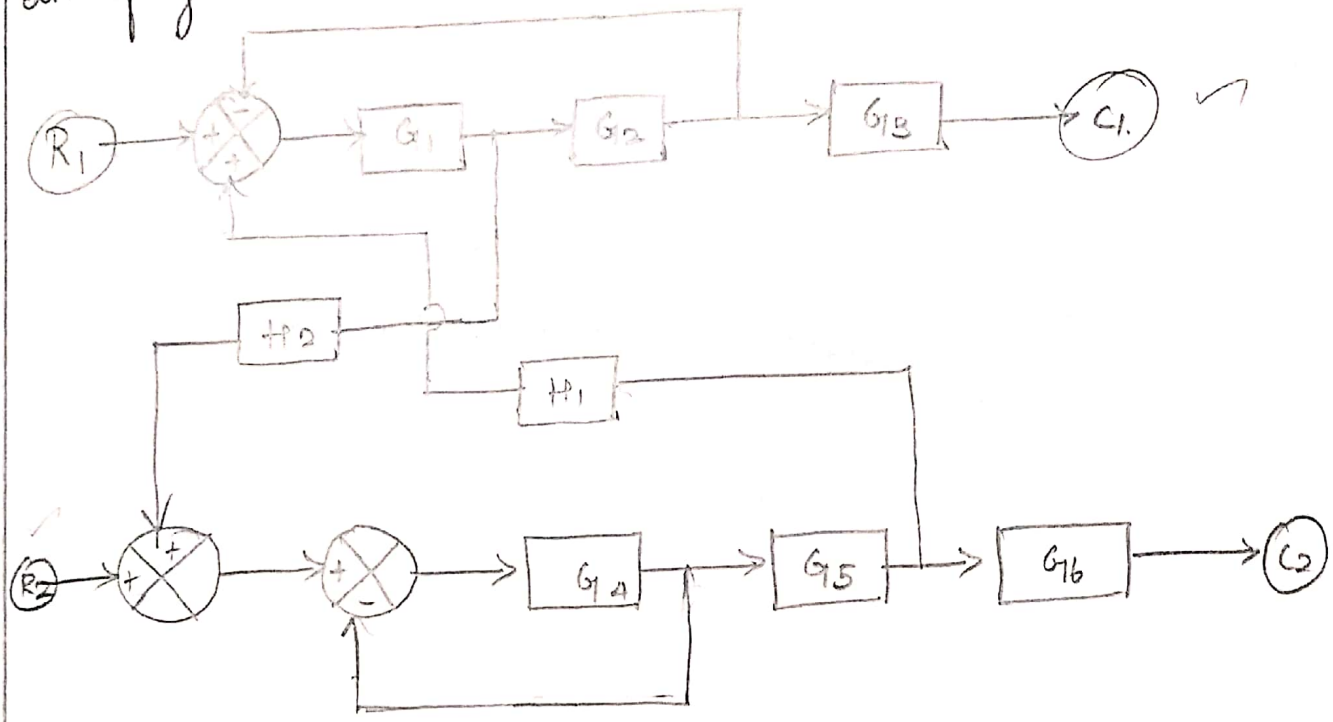
$$\frac{G}{1+GH} = \frac{\dfrac{G_1 G_3 G_4 G_2}{1 + G_3 G_4 H_1 + G_3 H_2}}{1 + G_4 \left(\dfrac{G_1 G_3 G_4 G_2}{1 + G_3 G_4 H_1 + G_3 H_2} \right)}$$

$$\frac{C}{R} = \frac{G_1 G_3 (G_4)^2 G_2}{1 + G_3 G_4 H_1 + G_3 H_2 + G_1 G_3 (G_4)^2}$$

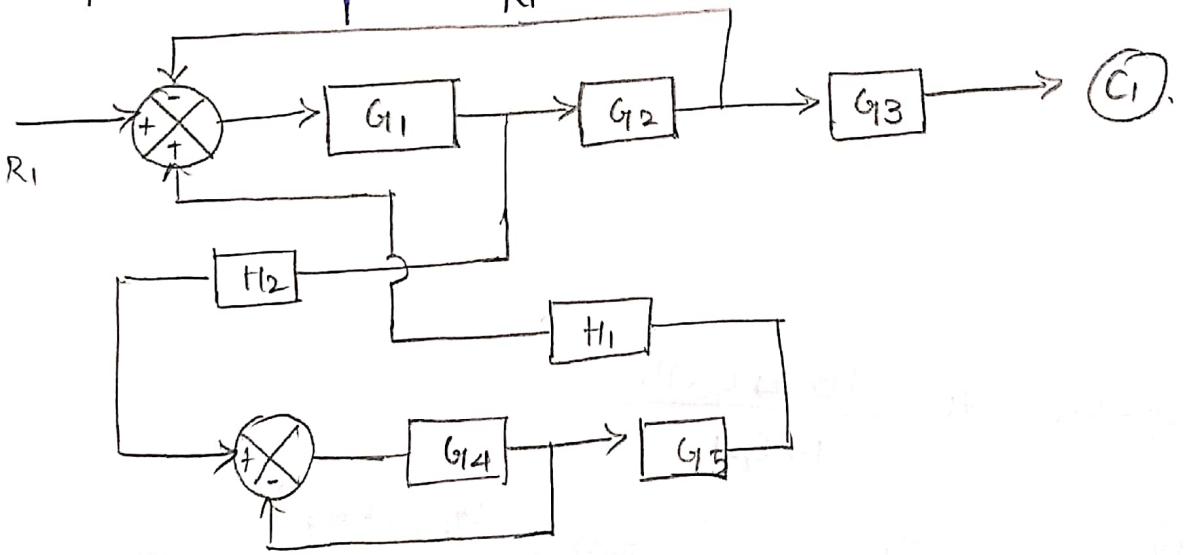
$$\frac{C}{R} = \frac{G_1 G_3 (G_4)^2 G_2}{1 + G_3 G_4 H_1 + G_3 H_2 + G_1 G_3 (G_4)^2}.$$

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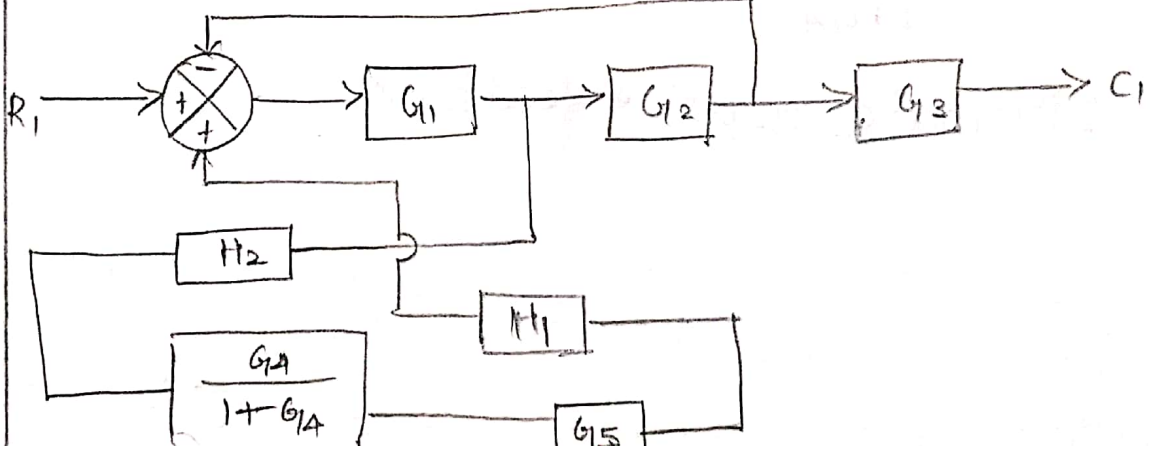
For the System Represented by Block Diagram Shown in figure to determine C_1/R_1 & C_2/R_1



Step 1 :- To find $\frac{C_1}{R_1}$ $R_2 = 0, C_2 = 0$.

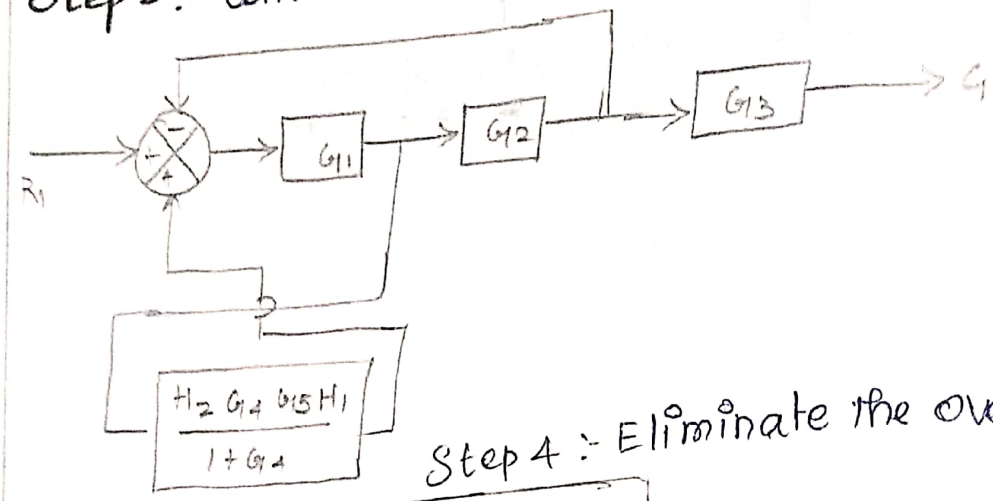


Step 2 :- $G = G_4$ $H = 1$ $\frac{G}{1+GH} = \frac{G_4}{1+G_4}$

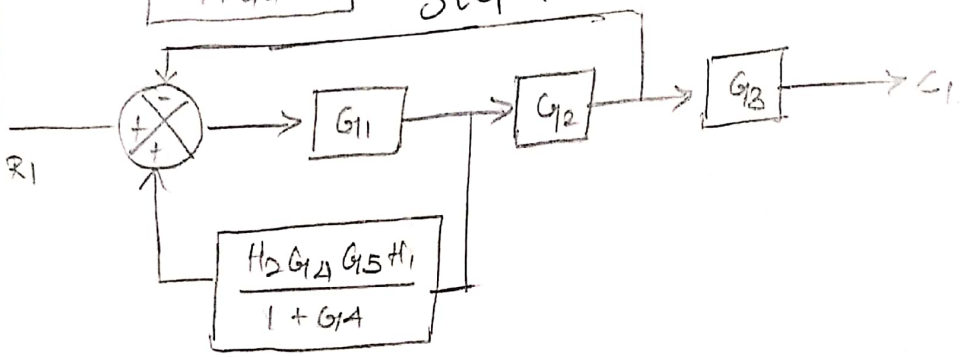


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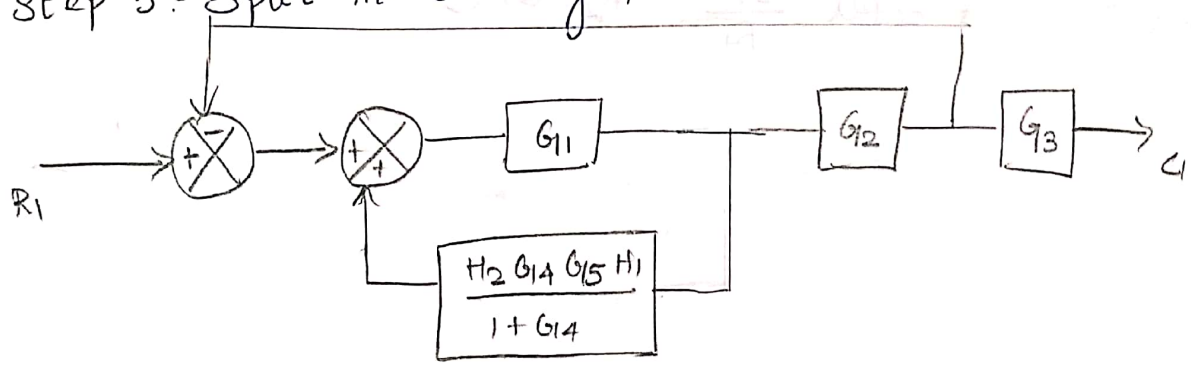
Step 3:- Combine the block in series.



Step 4:- Eliminate the over Loop



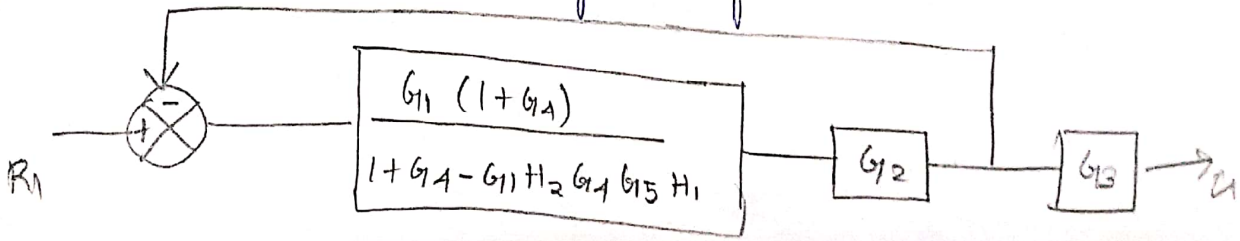
Step 5:- Split the summing point.



$$G = G_1 \;,\; H = \frac{H_2 G_4 G_5 H_1}{1+G_4}$$

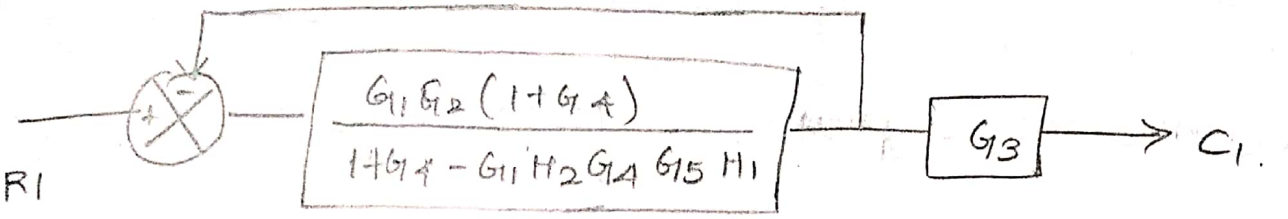
$$\frac{G}{1-GH} = \frac{G_1}{1-\frac{G_1 H_2 G_4 G_5 H_1}{1+G_4}} \Rightarrow \frac{G_1(1+G_4)}{1+G_4-G_1 H_2 G_4 G_5 H_1}$$

Step 6:- Eliminate negative feedback.



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Step 7 :- Combine the Block in Series.



$$G = \frac{G_1 G_2 (1+G_4)}{1+G_4 - G_1 H_2 G_4 G_5 H_1} \qquad H = 1.$$

$$\frac{G}{1+GH} = \frac{\dfrac{G_1 G_2 (1+G_4)}{1+G_4 - G_1 H_2 G_4 G_5 H_1}}{1 + \dfrac{G_1 G_2 (1+G_4)}{1+G_4 - G_1 H_2 G_4 G_5 H_1}}$$

$$= \frac{\dfrac{G_1 G_2 (1+G_4)}{\cancel{1+G_4 - G_1 H_2 G_4 G_5 H_1}}}{\dfrac{1+G_4 - G_1 H_2 G_4 G_5 H_1 + G_1 G_2 (1+G_4)}{1+G_4 - G_1 H_2 G_4 G_5 H_1}}$$

$$= \frac{G_1 G_2 + G_1 G_2 G_4 (G_3)}{1+G_4 - G_1 H_2 G_4 G_5 H_1 + G_1 G_2 + G_1 G_2 G_4}$$

$$\frac{C}{R} = \frac{G_1 G_2 G_3 + G_1 G_2 G_4 G_3}{1+G_4 - G_1 H_2 G_4 G_5 H_1 + G_1 G_2 + G_1 G_2 G_4}$$

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Signal flow graph:-

Elements in signal flow graph:

Node:- It Represent variable (or) signal.

Branch:- A Branch is directed Line Segment joining of two nodes.

Individual Loop:- Starting from a Node and Return come to same Node without crossing any node more than one.

Non-touching Loop:- If the Loop does not have a common node.

Procedure for signal flow graph:

Step 1: To find number of forward path.

Step 2: To find Number of Individual Loop.

Step 3: To find two non-touching Loop.

Step 4:- To find three non-touching Loop.

Step 5:- To find the value of Δ.

$\Delta = 1 - (\text{Sum of individual Loop}) + (\text{Sum of two non-touching Loop}) - (\text{Sum of three non-touching Loop})$.

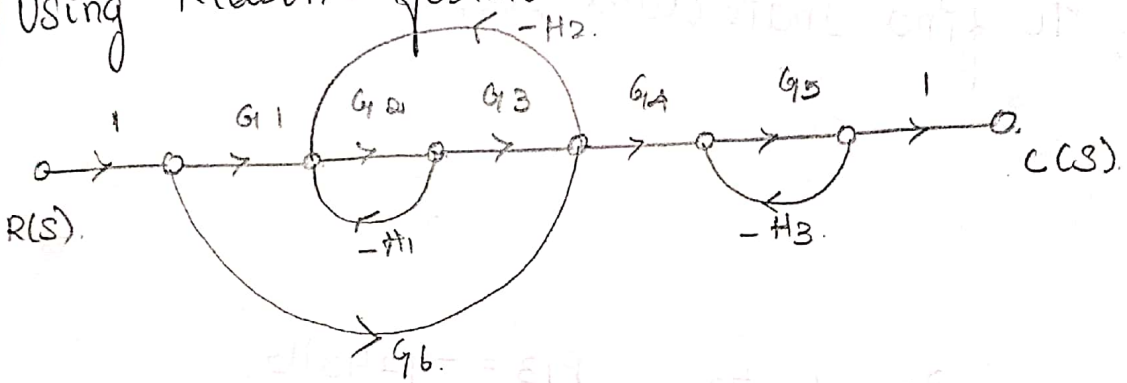
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Step 6 :- To find the Transfer Function by using Mason's gain formula.

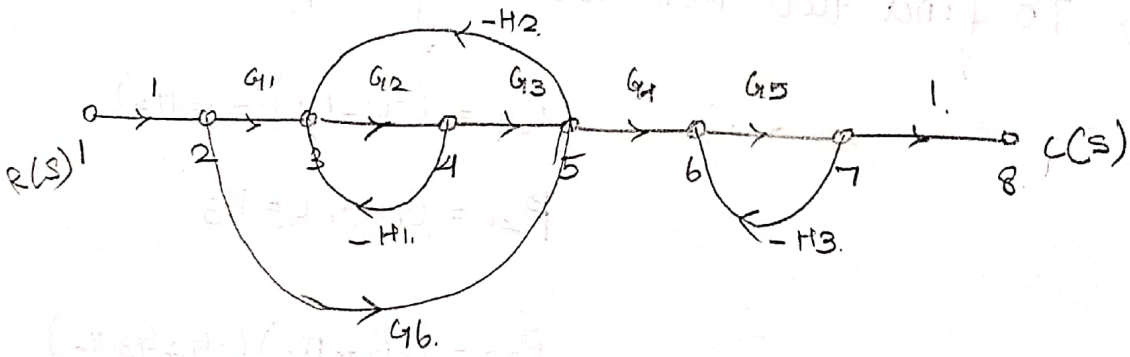
$$T.F = \frac{1}{\Delta} \sum P_K \Delta_K$$

K → No. of forward path.

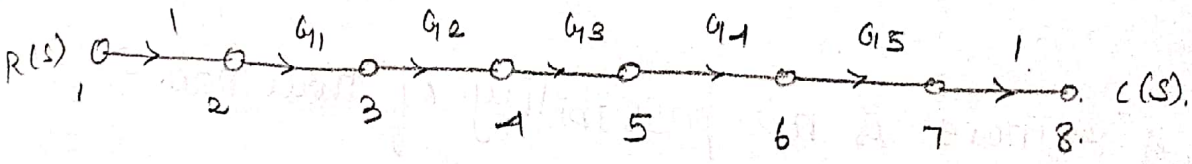
1) Find the transfer Function for the signal flow graph using Mason's formula.



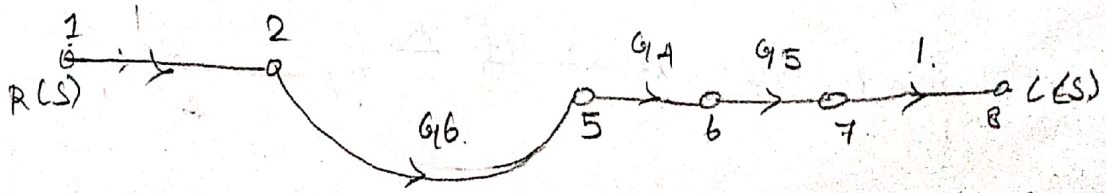
Put the Node Value:



Step 1 :- To find forward path.



$$P_1 = G_1 G_2 G_3 G_4 G_5$$



$$P_2 = G_6 G_4 G_5.$$

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K = 2

Mason's gain formula.

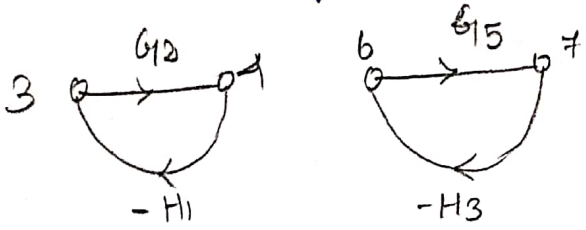
$\Delta, \Delta_1, \Delta_2 = ?$

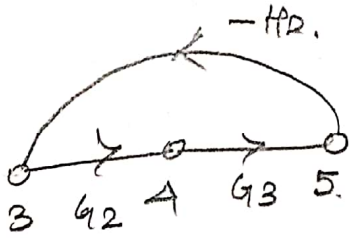
$P_1, P_{21} = ?$

$$T.F = \frac{1}{\Delta} \sum P_K \Delta_K$$

$$T.F = \frac{1}{\Delta} (P_1 \Delta_1 + P_2 \Delta_2).$$

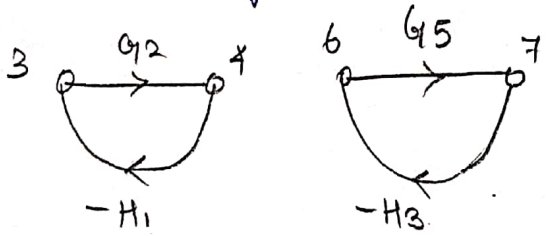
Step 2:- To find Individual Loop.





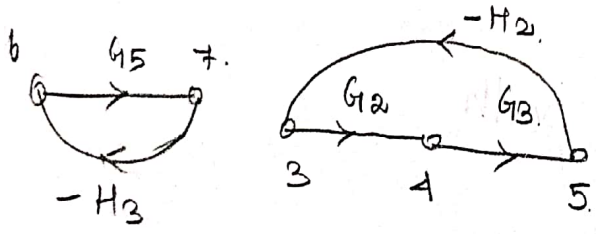
$P_{11} = -G_2 H_1$ $P_{12} = -G_5 H_3$ $P_{13} = -G_2 G_3 H_2.$

Step 3:- To find two non-touching Loop.



$P_{21} = (-G_2 H_1)(-G_5 H_3)$

$P_{21} = G_2 H_1 G_5 H_3.$



$P_{22} = (-G_5 H_3)(-G_2 G_3 H_2)$

$P_{22} = G_5 H_3 G_2 G_3 H_2.$

Step 4 :- There is no possibility of three non-touching Loop.

Step 5 :- To find the values of Δ

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Δ = 1 - (Sum of individual Loop) + (Sum of two non-touching Loop) - (Sum of three non-touching Loop).

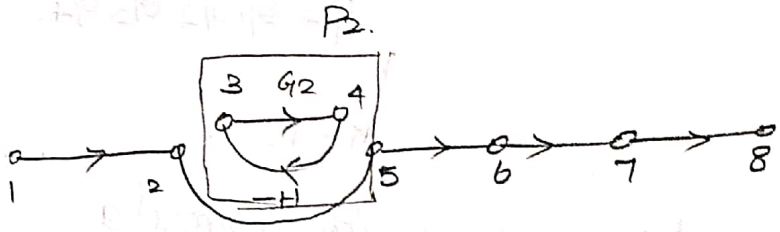
$\Delta = 1 - (P_1 + P_{12} + P_{13}) + (P_{21} + P_{22})$.

$= 1 - (-G_2H_1 - G_5H_3 - G_2G_3H_2) + (G_2H_1G_5H_3 + G_5H_3G_2G_3H_2)$

$\Delta = 1 + G_2H_1 + G_5H_3 + G_2G_3H_2 + G_2H_1G_5H_3 + G_5H_3G_2G_3H_2$.

Step 6 :- To find the Transfer Function.

$$T.F = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2)$$



$\Delta_1 = 1$

$\Delta_2 = 1 - (-G_2H_1)$.

$\Delta_2 = 1 + G_2H_1$.

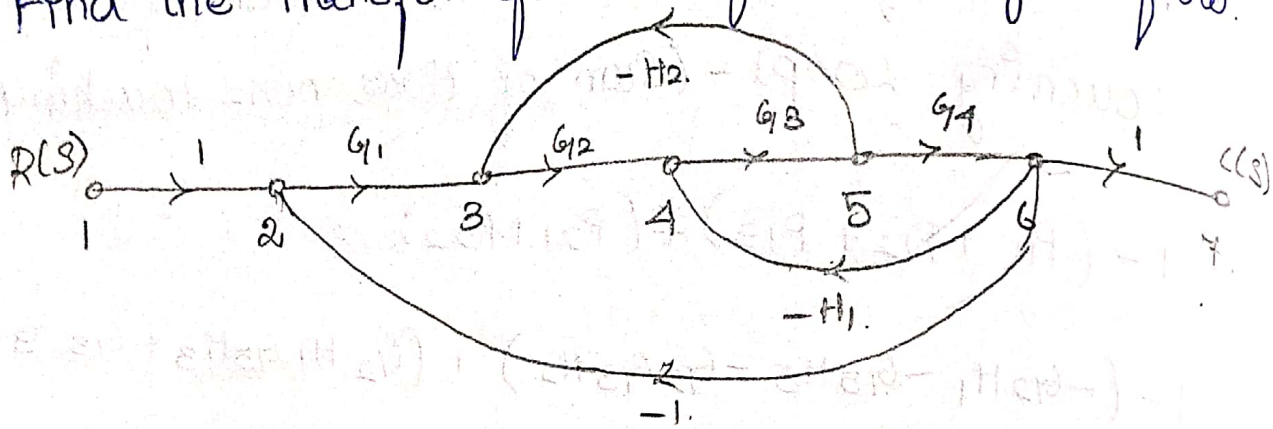
$$T.F = \frac{G_1G_2G_3G_4G_5(1) + G_6G_4G_5(1 + G_2H_1)}{1 + G_2H_1 + G_5H_3 + G_2G_3H_2 + G_2H_1G_5H_3 + G_5H_3G_2G_3H_2}$$

$$T.F = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

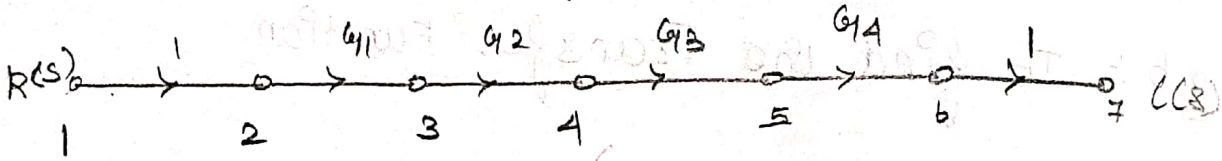
$$T.F = \frac{G_1G_2G_3G_4G_5 + G_6G_4G_5 + G_6G_4G_5G_2H_1}{1 + G_2H_1 + G_5H_3 + G_2G_3H_2 + G_2H_1G_5H_3 + G_5H_3G_2G_3H_2}$$

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2) Find the transfer function for the signal flow.



Step 1:- To find the forward path.



$$P_1 = G_1 G_2 G_3 G_4.$$

$$K = 1.$$

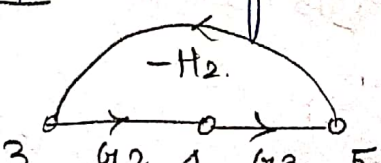
Mason's Gain formula.

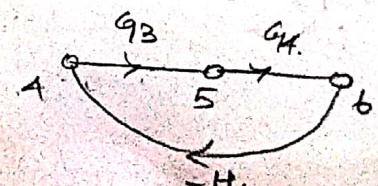
$$T.F = \frac{1}{\Delta} \sum P_K \Delta_K.$$

$$= \frac{1}{\Delta} (P_1 \Delta_1).$$

$\Delta = 1 -$ (Sum of Individual Loop) + (Sum of two non-touching Loop).

Step 2:- To find individual Loop.

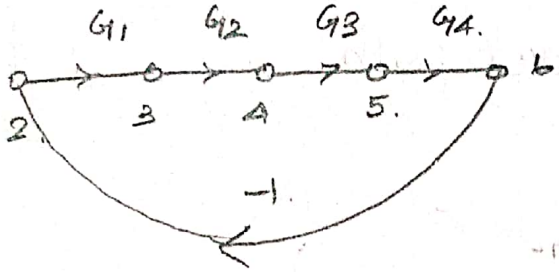




$P_{21} = -G_2 G_3 H_2$

$P_{22} = -G_3 G_4 H_1$

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$\Delta_1 = 1$

$P_{23} = -G_1 G_2 G_3 G_4.$

Step 3 :- There is no possibility of two and three non-touching Loop.

Step 4 :- To find the value of Δ

$$\Delta = 1 - (-G_2 G_3 H_2 - G_3 G_4 H_1 - G_1 G_2 G_3 G_4).$$

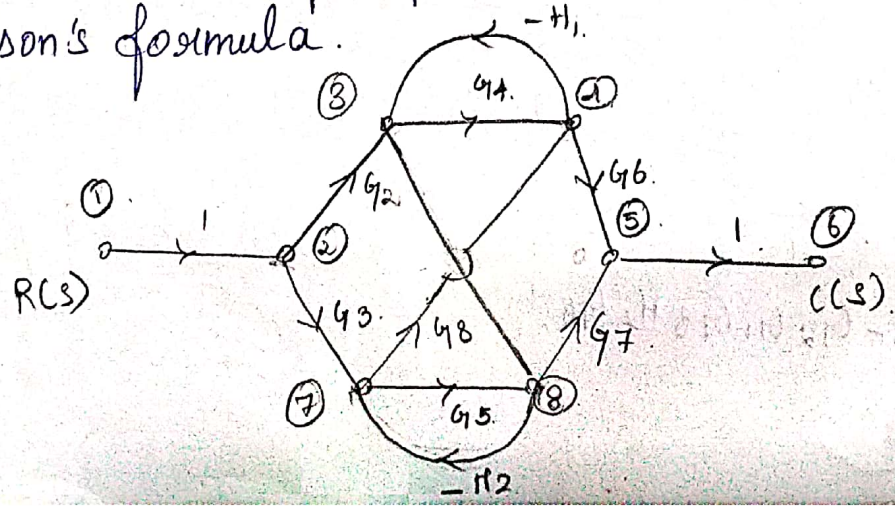
$$\Delta = 1 + G_2 G_3 H_2 + G_3 G_4 H_1 + G_1 G_2 G_3 G_4.$$

Step 5 :- To find the Transfer function.

$$T.F = \frac{1}{\Delta}(P_1 \Delta_1)$$

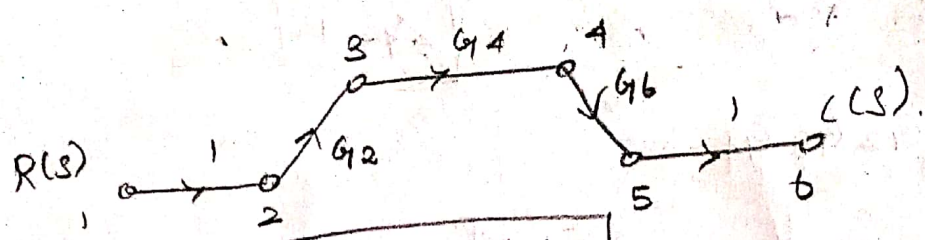
$$= \frac{P_1 \Delta_1}{\Delta} = \frac{G_1 G_2 G_3 G_4 (1).}{1 + G_2 G_3 H_2 + G_3 G_4 H_1 + G_1 G_2 G_3 G_4}$$

3) Find the Transfer function for signal diagram using Mason's formula.

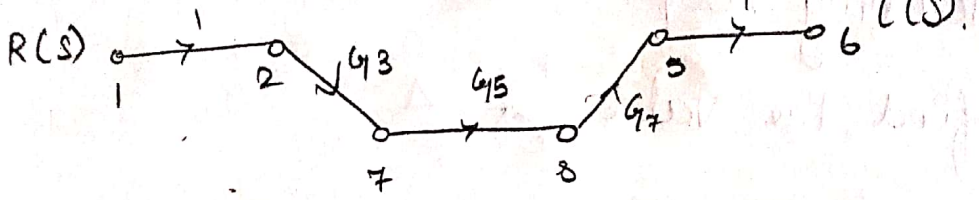


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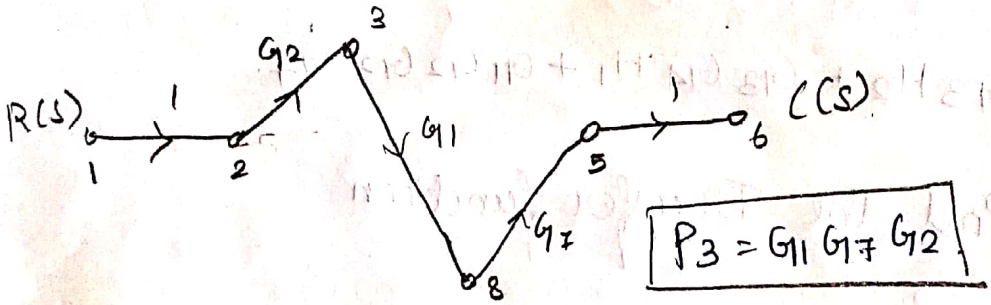
Step 1:- Number of forward path :-



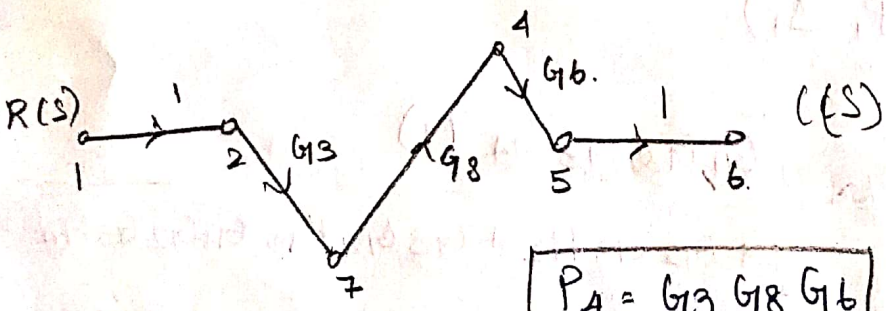
$P_1 = G_4 G_6 G_2$



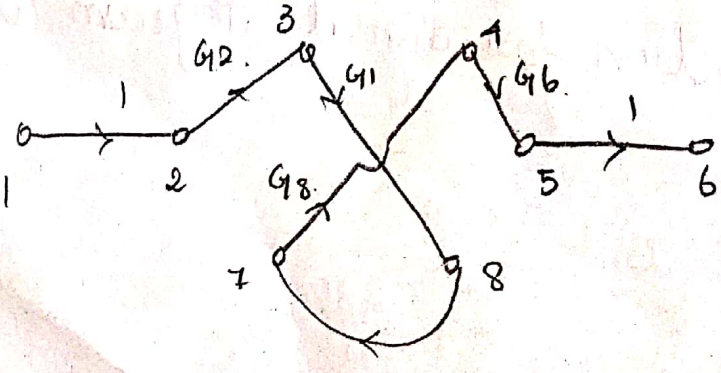
$P_2 = G_3 G_5 G_7$



$P_3 = G_1 G_7 G_2$

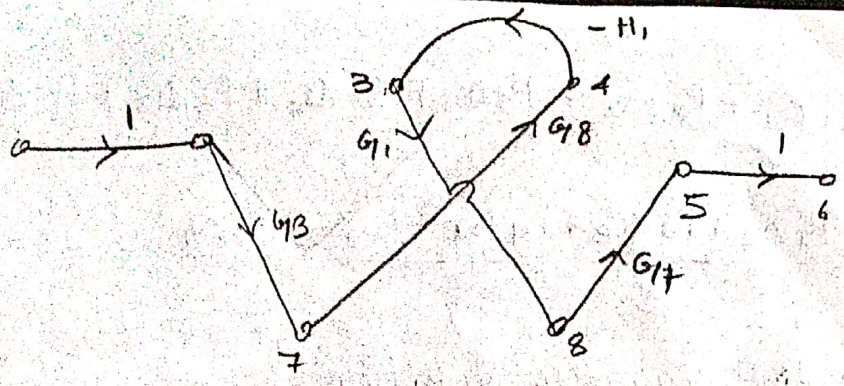


$P_4 = G_3 G_8 G_6$



$P_5 = -G_2 G_1 G_8 H_2 G_6$

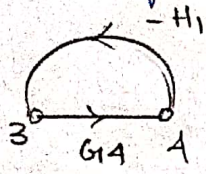
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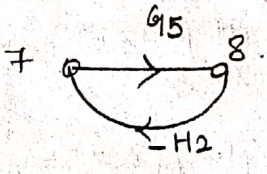


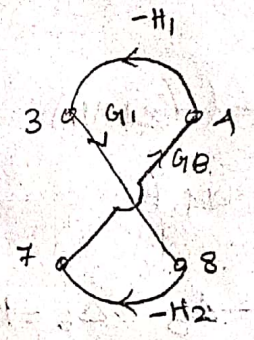
$$P_6 = -G_3 G_8 G_1 G_7 H_1$$

$\Delta = 1 - (\text{Sum of Individual Loop}) + (\text{Sum of two non-touching Loop})$.

Step 2:- To find Individual Loop.



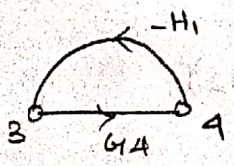


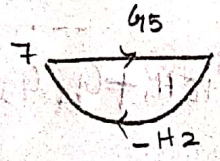


$P_{11} = -G_4 H_1$ $\quad P_{12} = -G_5 H_2$.

$P_{13} = G_1 H_2 G_8 H_1$.

Step 3:- To find two non-touching Loop.





$P_{21} = G_4 G_5 H_1 H_2$.

$\Delta = 1 - (P_{11} + P_{12} + P_{13}) + (P_{21})$.

$\Delta = 1 - (-G_4 H_1 - G_5 H_2 + G_1 H_2 G_8 H_1) + (G_4 G_5 H_1 H_2)$.

Using Mason's formula.

$$T.F = \frac{1}{\Delta} \sum P_K \Delta_K .$$

$K = 6$

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$$= \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5 + P_6\Delta_6)$$

$$T.F = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5 + P_6\Delta_6}{\Delta}$$

Step 4 :- To find Transfer function :-

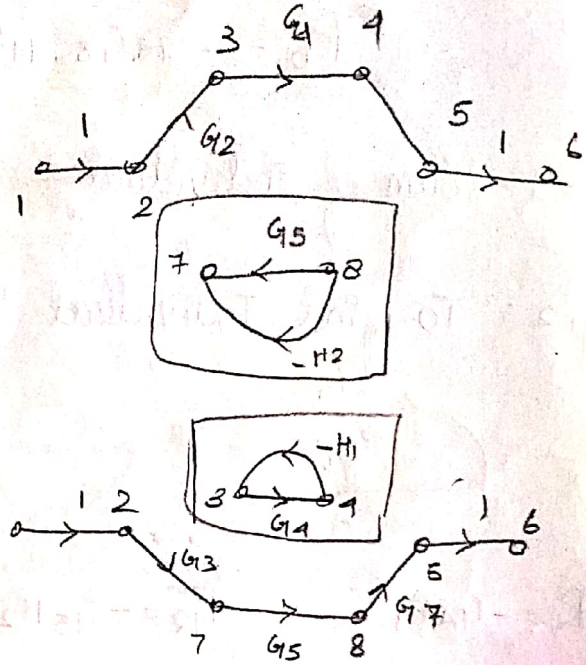
$\Delta_1 = 1 - (-G_5H_2)$.

$\Delta_1 = 1 + G_5H_2$.

$\Delta_2 = 1 - (-G_4H_1)$

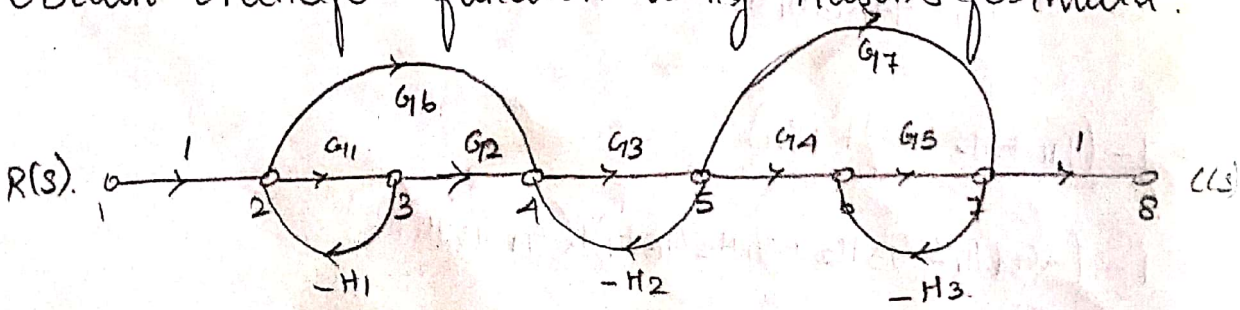
$\Delta_2 = 1 + G_4H_1$.

$\Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 1$.

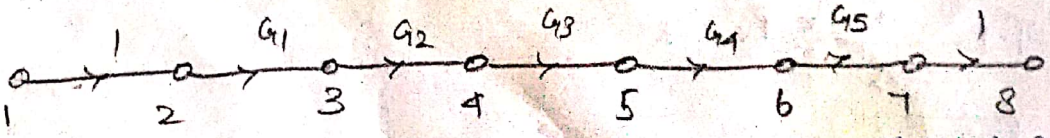


$$T.F = \frac{G_4G_6G_2(1+G_5H_2) + G_3G_5G_7(1+G_4H_1) + G_1G_7G_2(1) + G_3G_8G_6(1) + (-G_2G_1G_8H_2G_6)(1) + (-G_3G_8G_1G_7H_1)(1)}{(1 + G_4H_1 + G_5H_2 - G_1H_2G_8H_1 + G_4G_5H_1H_2)}$$

4. Obtain transfer function using Mason's formula.

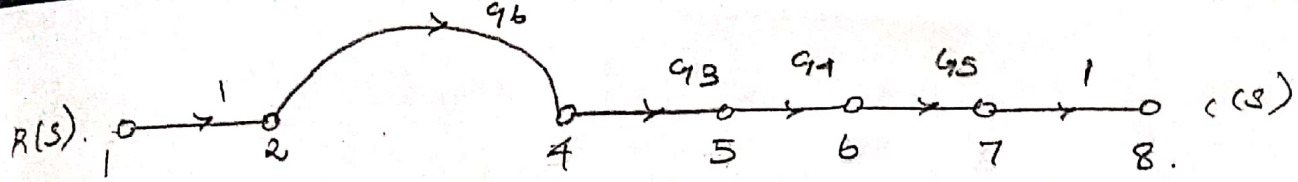


Step 1 :- To find number of forward path.

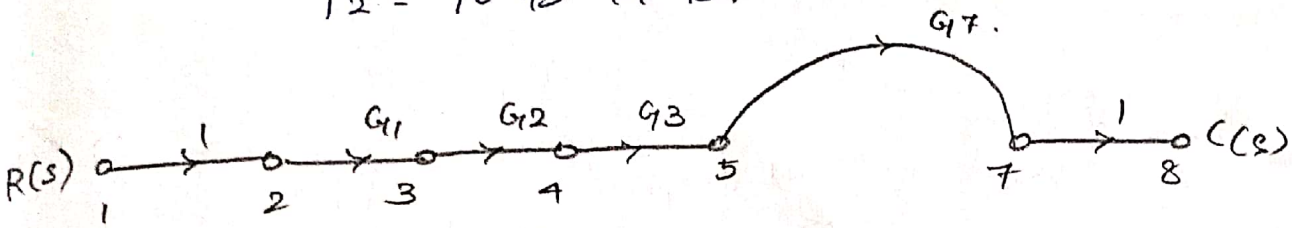


$P_1 = G_1G_2G_3G_4G_5$

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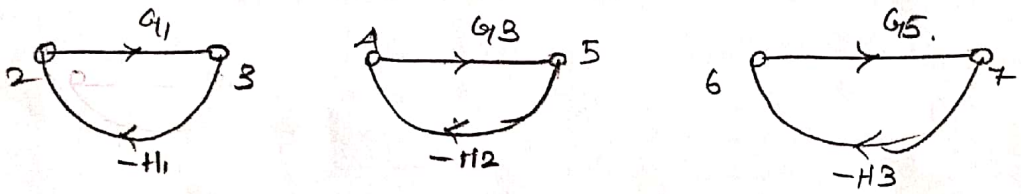


$P_2 = G_6 G_3 G_4 G_5$.



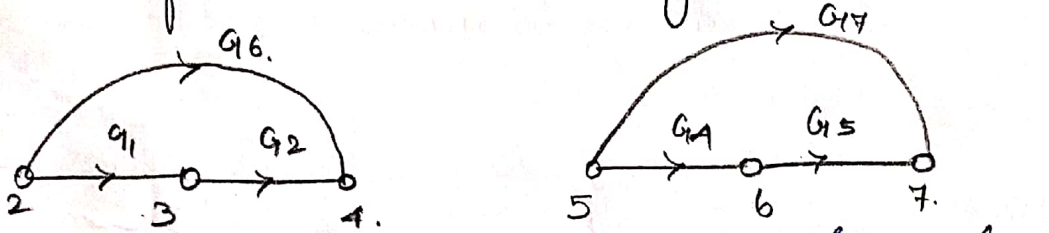
$P_3 = G_1 G_2 G_3 G_7$

Step 2:- To find individual loop.



$P_{11} = -G_1 H_1$ $P_{12} = -G_3 H_2$ $P_{13} = -G_5 H_3$.

Step 3:- To find two non-touching loop.



$= G_1 G_2 G_6$ $= G_4 G_5 G_7$

$$= \frac{1}{\Delta}(P_1 \Delta_1 + P_2 \Delta_2 + P_3 \Delta_3)$$

$\Delta = 1 - (\text{Sum of } \Sigma L) + (\text{Sum of two non-touching loop})$

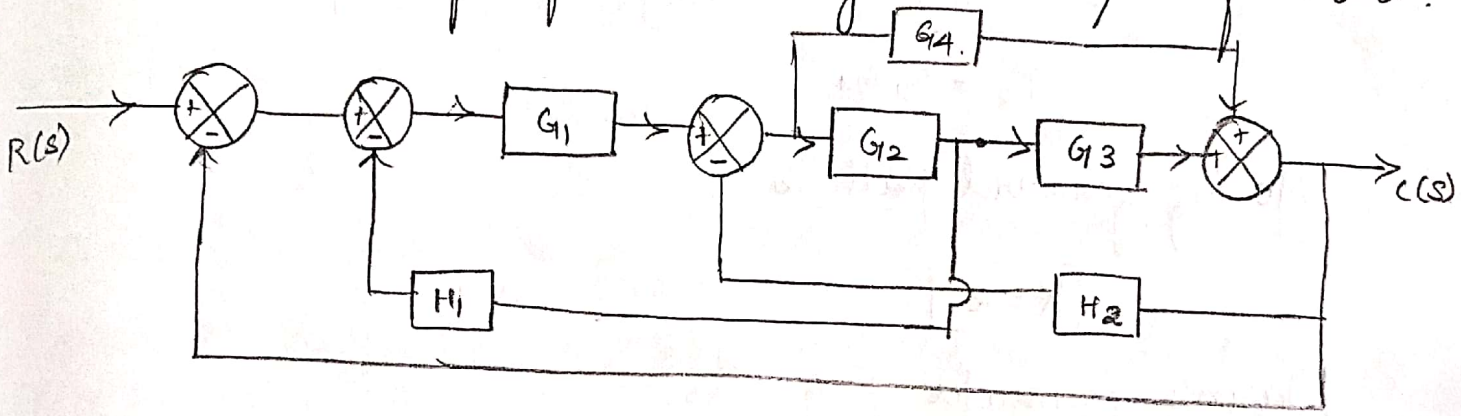
$= 1 - (-G_1 H_1 - G_3 H_2 - G_5 H_3) + (G_1 G_2 G_6 + G_4 G_5 G_7)$

$$\Delta = 1 + G_1 H_1 + G_3 H_2 + G_5 H_3 + G_1 G_2 G_6 + G_4 G_5 G_7$$

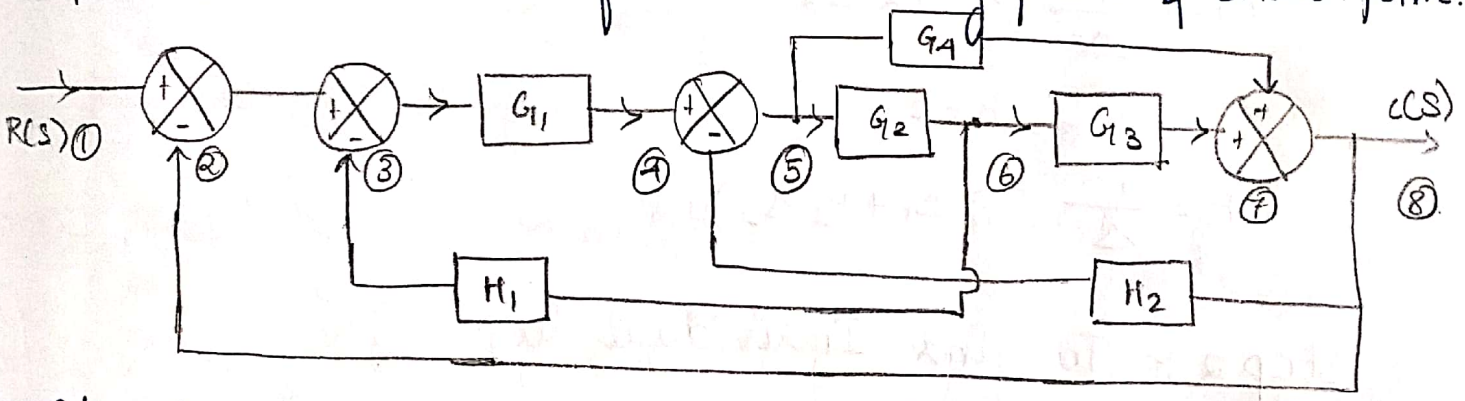
$$T.F = \frac{G_1 G_2 G_3 G_4 G_5 \Delta + G_6 G_3 G_4 G_5 + G_1 G_2 G_3 G_7}{1 + G_1 H_1 + G_3 H_2 + G_5 H_3 + G_1 G_2 G_6 + G_4 G_5 G_7}$$

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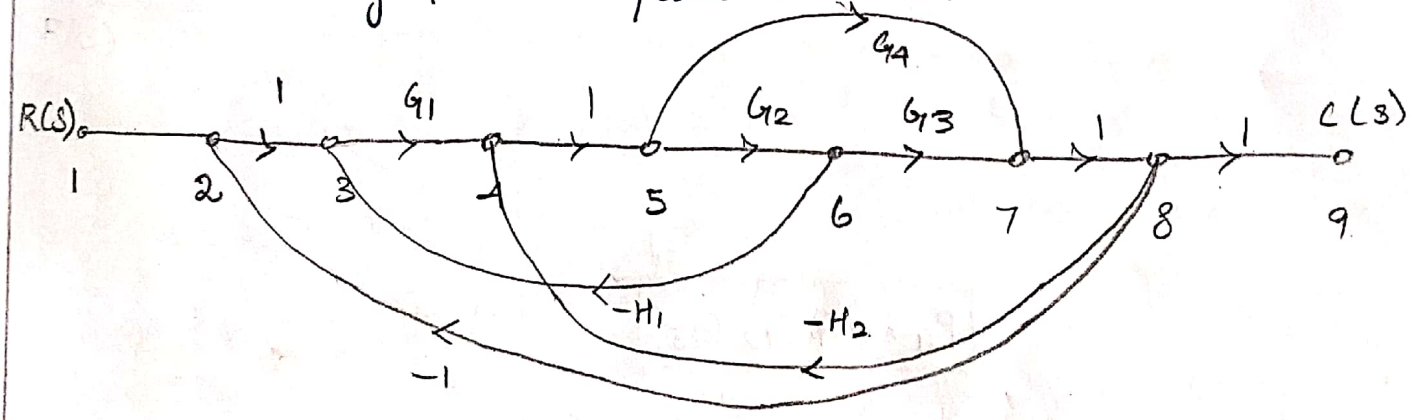
1) Convert Block diagram to Signal flow graph and determine the transfer function using Mason's Gain formula.



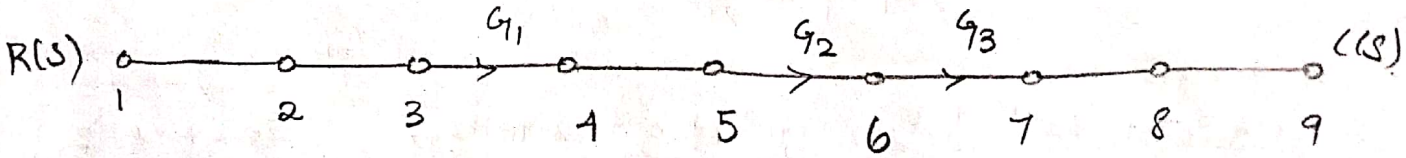
Step 1:- Give node value for Each Summing point & Branch point.



Consider Summing & Branch point as node.



Step 1:- To find forward path.



$$P_1 = G_1 G_2 G_3$$

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$$P_2 = G_1 G_4.$$

No. of forward path 2

$$K = 2$$

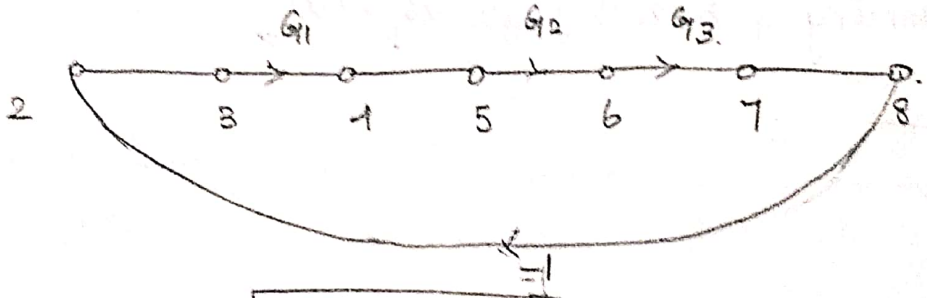
Mason's formula

$$T.F = \frac{1}{\Delta} \sum P_K \Delta_K$$

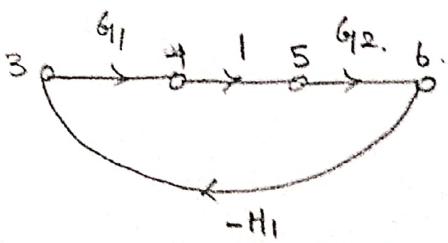
$$K = 2$$

$$T.F = \frac{1}{\Delta} (P_1 \Delta_1 + P_2 \Delta_2).$$

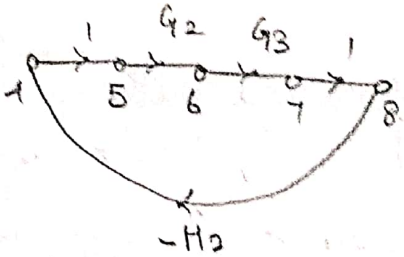
Step 2 :- To find Individual loop :



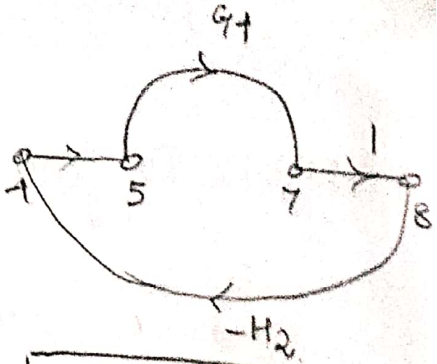
$$P_{11} = -G_1 G_2 G_3.$$



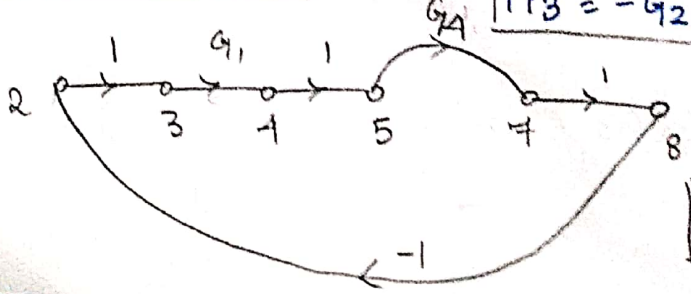
$$P_{12} = -G_1 G_2 H_1$$



$$P_{13} = -G_2 G_3 H_2$$



$$P_{14} = -G_4 H_2.$$



$$P_{15} = -G_1 G_4.$$

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Step 3:- To find two non-touching Loop:

There is no posibility to take two non-touching loop.

Step 4:- To find the value of Δ

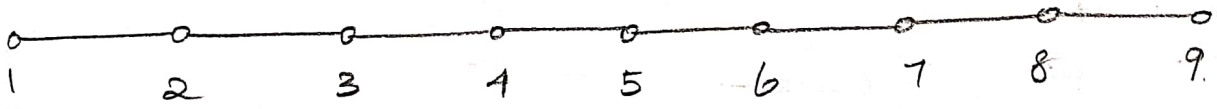
$\Delta = 1 - (\text{Individual Loop}) + (\text{Sum of two non-touching Loop})$

$\Delta = 1 - (-G_1G_2G_3 - G_4H_2 - G_1G_2H_1 - G_2G_3H_2 - G_1G_4) + (0)$

$$\boxed{\Delta = 1 + G_1G_2G_3 + G_4H_2 + G_1G_2H_1 + G_2G_3H_2 + G_4G_1}$$

Step 5:- To find Δ_1 & Δ_2

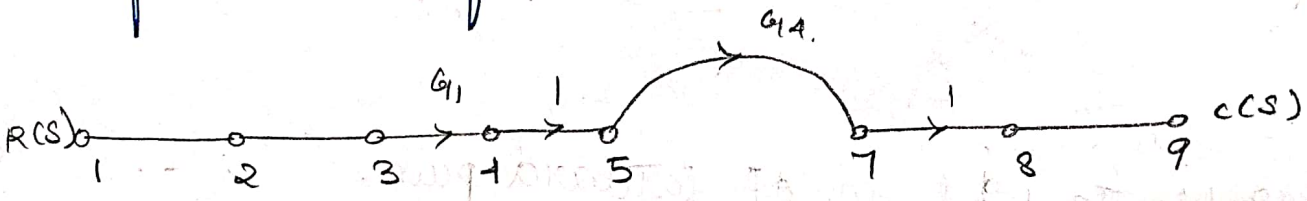
To find Δ_1 in forward path 1



There is no possible to take one Loop without touch the forward path 1

So, $\Delta_1 = 1$

To find Δ_2 in forward path 2.



$\Delta_2 = 1$

Mason's formula

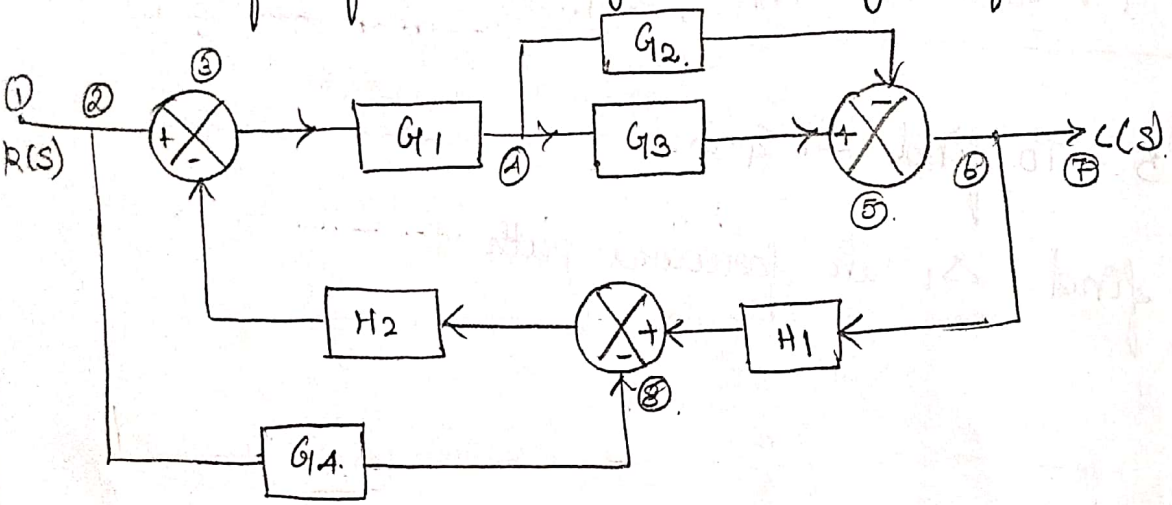
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$$T.F = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2)$$

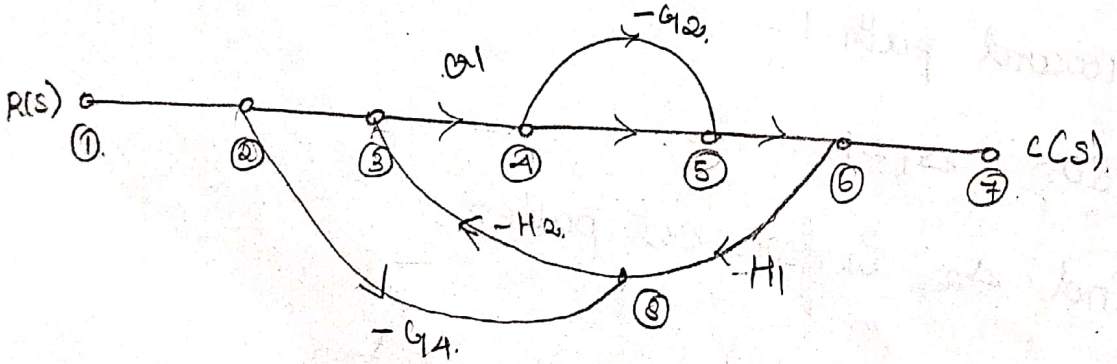
$$= \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$T.F = \frac{G_1G_2G_3(1) + G_1G_4(1)}{1 + G_1G_2G_3 + G_1G_2H_1 + G_2G_3H_2 + G_1G_4 + G_4H_2}$$

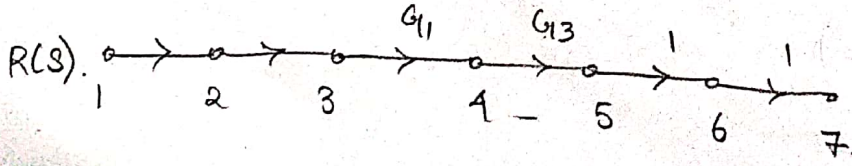
2) Draw a Signal flow graph and Evaluate the closed Loop Transfer function using Mason's gain formula.



Consider Summing & Branch point as node.

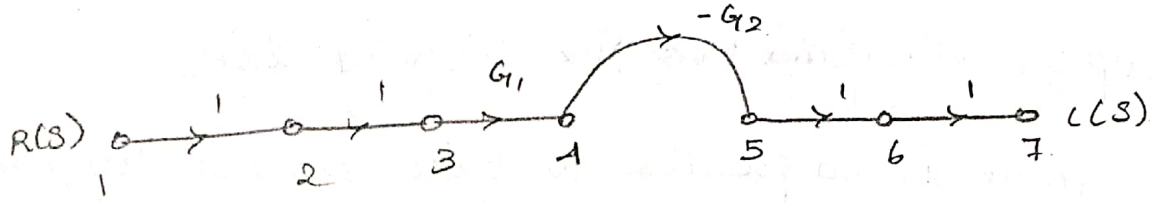


Step 1:- To find no. of forward path.

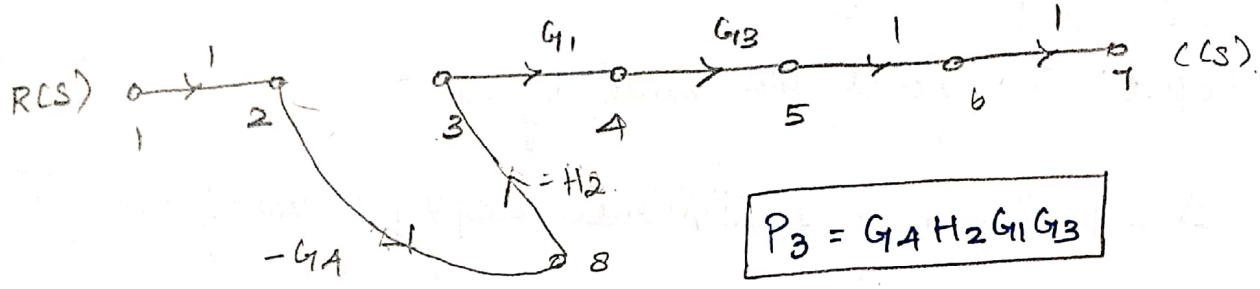


$P_1 = G_1G_3$

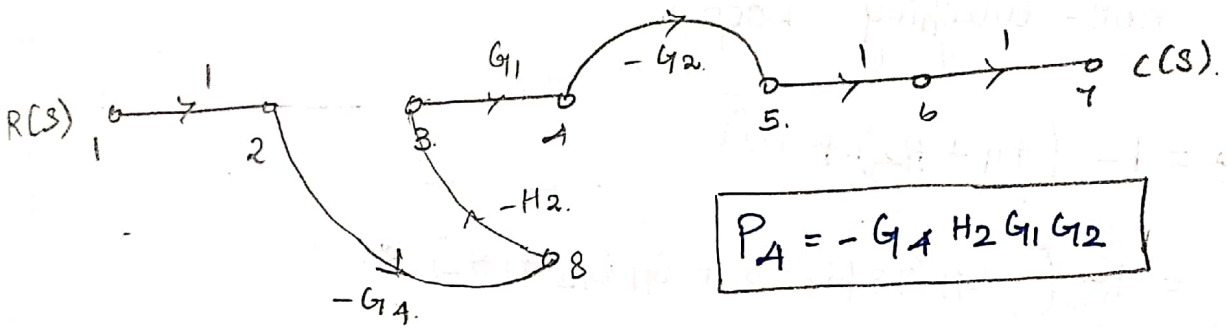
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$$P_2 = -G_1 G_2$$



$$P_3 = G_4 H_2 G_1 G_3$$



$$P_4 = -G_4 H_2 G_1 G_2$$

No. of forward path 4

$K = 4$

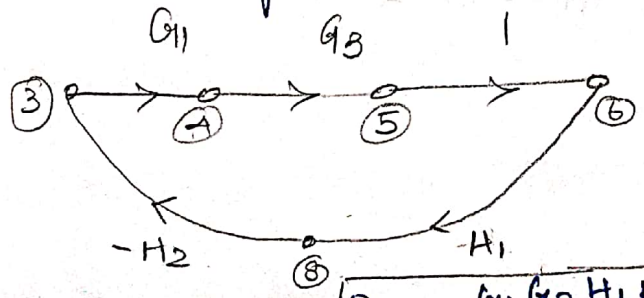
Mason's formula

$$T.F = \frac{1}{\Delta} \sum P_K \Delta_K$$

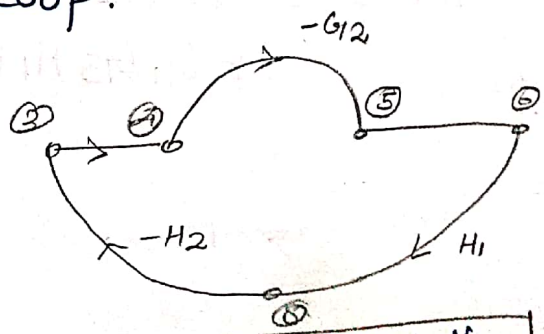
$K = 4$.

$$T.F = \frac{1}{\Delta} (P_1 \Delta_1 + P_2 \Delta_2 + P_3 \Delta_3 + P_4 \Delta_4).$$

Step 2 :- To find Individual Loop.



$$P_{11} = -G_1 G_3 H_1 H_2$$



$$P_{12} = G_1 G_2 H_1 H_2.$$

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Step 3 :- To find two Non-touching Loop

There is no possible to take two non-touching Loop.

Step 4 :- To find the value of Δ

$\Delta = 1 - (\text{Sum of Individual Loop}) + (\text{Sum of two non-touching Loop.})$

$\Delta = 1 - (P_{11} + P_{12}) + (0)$

$= 1 - (-G_1 G_3 H_1 H_2 + G_1 G_2 H_1 H_2).$

$$\boxed{\Delta = 1 + G_1 G_3 H_1 H_2 - G_1 G_2 H_1 H_2.}$$

Step 5 :- To find $\Delta_1, \Delta_2, \Delta_3, \Delta_4$.

$$T.F = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4).$$

$$T.F = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4}{\Delta}.$$

$\Delta_1 = 1, \quad \Delta_2 = 1, \quad \Delta_3 = 1, \quad \Delta_4 = 1.$

$$T.F = \frac{G_1G_3 - G_1G_2 + G_4H_2G_1G_3 + (-G_4H_2G_1G_2)}{1 + G_1G_3H_1H_2 - G_1G_2H_1H_2}.$$

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Mathematical Model of control Systems:

i) Mechanical Translational System.

ii) Mechanical rotational System.

iii) Electrical Analogy of Mechanical Translational System.

iv) Electrical Analogy of Mechanical Rotational System.

i) Mechanical Translational System:

$$\text{Transfer Function} = \frac{\text{Laplace Transform of output.}}{\text{Laplace Transform of Input.}}$$

The Model of Mechanical Translational System can be obtained by using three basic Elements

i) Mass, ii) Dash-pot iii) Spring.

* The weight of the Mechanical System is represented by the element Mass.

* The Elastic deformation of the body can be represented by Spring

* The Friction Existing in Rotating Mechanical System can be Represented by dash-Pot

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List of Symbols used in Mechanical System.

x = Displacement, m

$V = \frac{dx}{dt}$ = Velocity, m/Sec

$a = \frac{d^2x}{dt^2}$ = Acceleration, m/Sec^2

f = force Applied, N (Newtons)

f_m = opposing force due to Mass (M)

f_B = opposing force due to dash-Pot (B)

f_K = opposing force due to Spring (K)

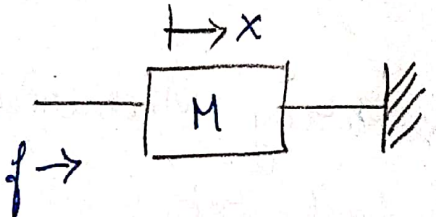
M = Mass, K = Spring, B = Dash-Pot.

Newtons Second Law:-

Sum of applied force is equal to Sum of opposing on the body.

Force-Balance Equation of Idealized elements:

i) Consider a Ideal Mass which is proportional to the acceleration.



$f_m \propto \frac{d^2x}{dt^2}$ (or) $f_m = \frac{Md^2x}{dt^2}$

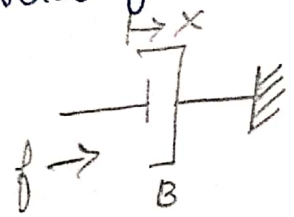
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Newtons Second Law:

Applied force = opposing force

$$f = f_m = M\frac{d^2x}{dt^2}$$

ii) Consider a Dash pot which is proportional to the velocity



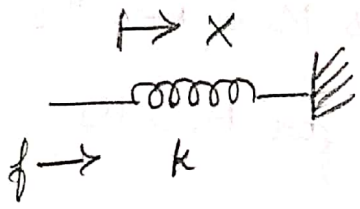
$$f_B \propto \frac{dx}{dt} \text{ (or) } f_B = B\frac{dx}{dt}$$

Applied force = opposing force

$$f = f_B = B\frac{dx}{dt}$$

iii) Consider as Spring which is proportional to the displacement.

a)

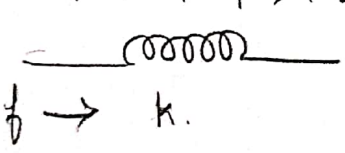


$$f_K \propto x \text{ (or) } f_K = Kx$$

Applied force = opposing force

$$f = f_K = Kx$$

b)

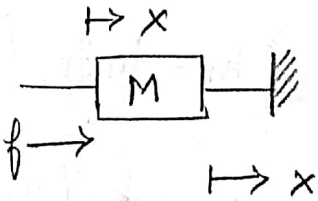


$$f_K = K(x_1 - x_2)$$

Applied force = opposing force

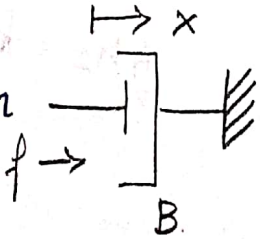
$$f = f_K = K(x_1 - x_2)$$

1) Mass → weight



$$f_m = M\frac{d^2x}{dt^2} \text{ (acceleration)}$$

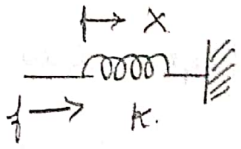
2) Dash pot → Friction



$$f_B = B\frac{dx}{dt} \text{ (Velocity)}$$

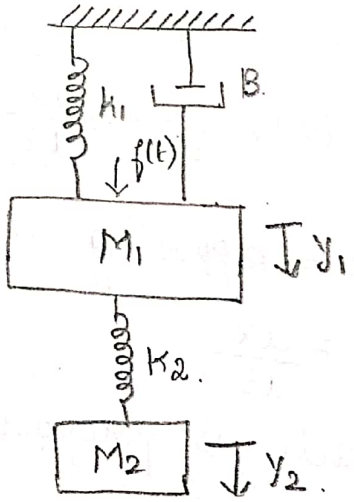
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3. Spring → Elastic deformation



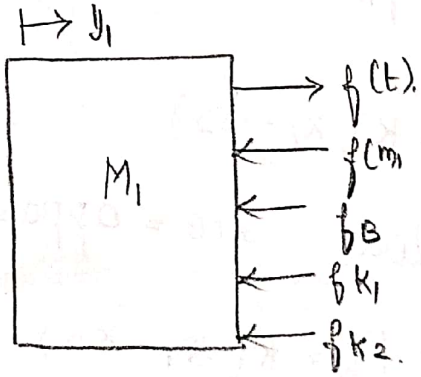
$f_K = kx$ (displacement)

1. Determine the transfer function $\frac{Y_2(s)}{F(s)}$ for the Mechanical Translational System as shown in figure.



Soln:- The System consists of two nodes M_1 & M_2. The governing differential Equation from for M_1 & M_2

Node M_1:



$f_{m_1} = M_1 \frac{d^2y_1}{dt^2}$ $\qquad$ $f_{K_1} = K_1 y_1$ (Displacement).

$f_B = B \frac{dy_1}{dt}$ $\qquad$ $f_{K_2} = K_2(y_1 - y_2)$

(velocity)

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Newton's Second Law:

Applied force = opposing force.

$$f(t) = f_{m_1} + f_B + f_{K_1} + f_{K_2}.$$

$$f(t) = M_1 \frac{d^2y_1}{dt^2} + B\frac{dy_1}{dt} + K_1 y_1 + K_2(y_1 - y_2) \quad \text{---} \quad (1)$$

Take Laplace transform for above equation.

Laplace transform $L.T\{f(t)\} = F(s)$

$$L.T\{y_1\} = y_1(s)$$

$$L.T\{y_2\} = y_2(s)$$

$$L.T\left\{\frac{d^2}{dt^2}\right\} = s^2$$

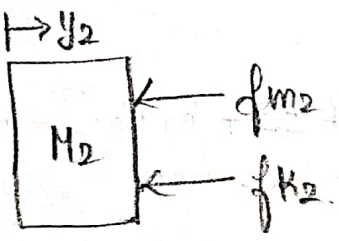
$$L.T\{d/dt\} = s.$$

Substitute Laplace transform in equation (1).

$$F(s) = M_1 s^2 y_1(s) + B s y_1(s) + K_1 y_1(s) + K_2 y_1(s) - K_2 y_2(s)$$

$$F(s) = y_1(s)\left[M_1 s^2 + Bs + K_1 + K_2\right] - K_2 y_2(s) \quad \text{---} \quad (2).$$

Node M_2:



Applied force = opposing force

$$0 = f_{m_2} + f_{K_2}.$$

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$$f_{m_2} = M_2 \frac{d^2 y_2}{dt^2}$$

$$f_{K_2} = K_2 (y_2 - y_1)$$

$$0 = M_2 \frac{d^2 y_2}{dt^2} + K_2 (y_2 - y_1)$$

Take Laplace transform for above equation.

$$0 = M_2 s^2 y_2(s) + K_2 [y_2(s) - y_1(s)]$$

$$0 = M_2 s^2 y_2(s) + K_2 y_2(s) - K_2 y_1(s)$$

$$0 = y_2(s)[M_2 s^2 + K_2] - K_2 y_1(s)$$

$$K_2 y_1(s) = y_2(s)[M_2 s^2 + K_2]$$

$$y_1(s) = \frac{y_2(s)[M_2 s^2 + K_2]}{K_2} \quad \text{---} \quad (3)$$

Sub equation (3) in equation (2).

$$F(s) = y_2(s) \frac{[M_2 s^2 + K_2]}{K_2} [M_1 s^2 + Bs + K_1 + K_2] - K_2 y_2(s)$$

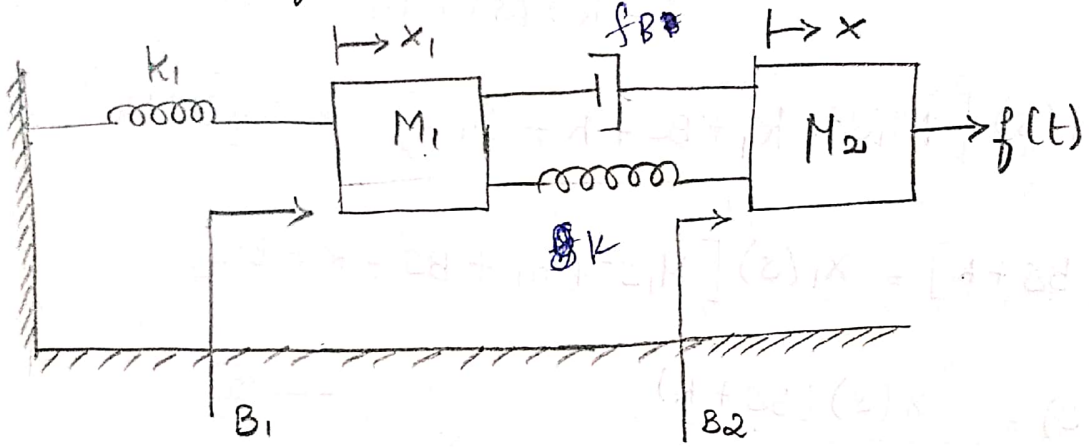
$$F(s) = \frac{y_2(s) [M_2 s^2 + K_2][M_1 s^2 + Bs + K_1 + K_2] - K_2^2 y_2(s)}{K_2}$$

$$F(s) = y_2(s) \frac{\{[M_2 s^2 + K_2][M_1 s^2 + Bs + K_1 + K_2] - K_2^2\}}{K_2}$$

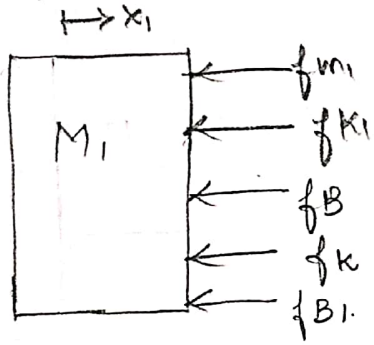
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$$\frac{Y_2(s)}{F(s)} = \frac{K_2}{[M_2 s^2 + K_2][M_1 s^2 + Bs + K_1 + K_2] - K_2^2}$$

2) Determine the transfer function $\frac{X(s)}{F(s)}$ for the Mechanical Translational System as shown.



Node M_1:



$$f_{m_1} = M_1 \frac{d^2 x_1}{dt^2}$$

$$f_{K_1} = K_1 x_1$$

$$f_B = B \frac{d}{dt}(x_1 - x) \qquad f_{B_1} = B_1 \frac{dx_1}{dt}$$

$$f_k = K(x_1 - x)$$

Applied force = opposing force

$$0 = f_{m_1} + f_{K_1} + f_B + f_k + f_{B_1}$$

$$0 = M_1 \frac{d^2 x_1}{dt^2} + K_1 x_1 + B \frac{d}{dt}(x_1 + x) + K(x_1 - x) + B_1 \frac{dx_1}{dt}$$

(1)

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Take Laplace transform for above eqn.

$$0 = M_1 s^2 X_1(s) + K_1 X_1(s) + Bs\left[X_1(s) - X(s)\right] + K\left[X_1(s) - X(s)\right] + B_1 s X_1(s)$$

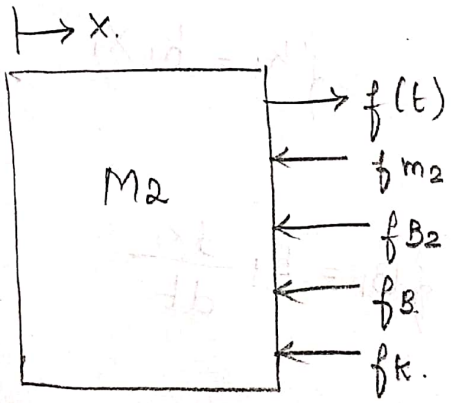
$$0 = M_1 s^2 X_1(s) + K_1 X_1(s) + Bs X_1(s) - Bs X(s) + K X_1(s) - K X(s) + B_1 s X_1(s)$$

$$0 = X_1(s)\left[M_1 s^2 + K_1 + Bs + K + B_1 s\right] - X(s)\left[Bs + K\right]$$

$$X(s)\left[Bs + K\right] = X_1(s)\left[M_1 s^2 + K_1 + Bs + K + B_1 s\right]$$

$$X_1(s) = \frac{X(s)(Bs + K)}{\left(M_1 s^2 + K_1 + Bs + K + B_1 s\right)} \quad \text{— (2)}$$

For Node M2 :



Applied force = opposing force.

$$f(t) = f_{m_2} + f_{B_2} + f_B + f_K$$

$$f_{m_2} = M_2 \frac{d^2x}{dt^2}$$

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$$f_{B_2} = B_2 \frac{dx}{dt}$$

$$f_B = B \frac{d}{dt}(x - x_1)$$

$$f_k = k(x - x_1).$$

$$f(t) = M_2 \frac{d^2x}{dt^2} + B_2 \frac{dx}{dt} + B \frac{d}{dt}(x - x_1) + k(x - x_1).$$

Take Laplace transform for above equation.

$$F(s) = M_2 s^2 X(s) + B_2 s X(s) + B s [X(s) - X_1(s)] + k[X(s) - X_1(s)]$$

$$F(s) = M_2 s^2 X(s) + B_2 s X(s) + Bs X(s) - Bs X_1(s) + k X(s) - k X_1(s)$$

$$F(s) = X(s)[M_2 s^2 + B_2 s + Bs + k] - X_1(s)[Bs + k].$$

Sub eqn ② in ③.

$$F(s) = X(s)[M_2 s^2 + B_2 s + Bs + k] - \frac{X(s)(Bs + k)}{(M_1 s^2 + K_1 + Bs + k + B_1 s)} (Bs + k).$$

$$F(s) = \frac{X(s)[M_2 s^2 + B_2 s + Bs + k][M_1 s^2 + K_1 + Bs + k + B_1 s] - X(s)(Bs + k)}{M_1 s^2 + K_1 + Bs + k + B_1 s}$$

$$F(s) = X(s)\left\{\frac{[M_2 s^2 + B_2 s + Bs + k][M_1 s^2 + K_1 + Bs + k + B_1 s] - (Bs + k)^2}{M_1 s^2 + K_1 + Bs + k + B_1 s}\right\}$$

$$\frac{X(s)}{F(s)} = \frac{M_1 s^2 + K_1 + Bs + k + B_1 s}{[M_2 s^2 + B_2 s + Bs + k][M_1 s^2 + K_1 + Bs + k + B_1 s] - (Bs + k)^2}$$

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Mechanical Rotational System:

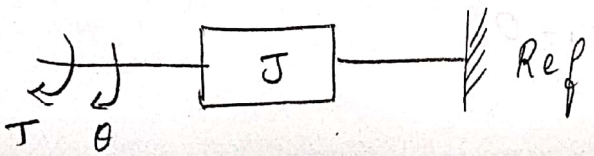
The elements are in Rotational they are

i) Moment of Inertia [J], Dashpot [B], Spring [K].

Mechanical Translational	Mechanical Rotational.
i) x = displacement	θ = displacement
ii) $\frac{dx}{dt}$ = velocity	$\frac{d\theta}{dt}$ = Angular velocity.
iii) $\frac{d^2x}{dt^2}$ = Acceleration	$\frac{d^2\theta}{dt^2}$ = Acceleration
iv) $F(t)$ = Applied force	T = Applied Torque.
v) F_m - opposing force due to Mass	T_j = opposing Torque due to Moment of inertia.
vi) F_B - Opposing force due to Dashpot.	T_B = opposing Torque due to Dashpot
vii) F_K - opposing force due to Spring.	T_K = opposing Torque due to Spring.

Force - Balance Equation for Mechanical Rotational System:-

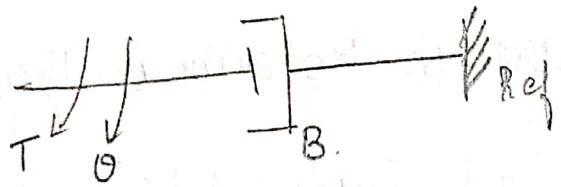
i) Moment of Inertia [J]



$$T = T_j = J\frac{d^2\theta}{dt^2}$$

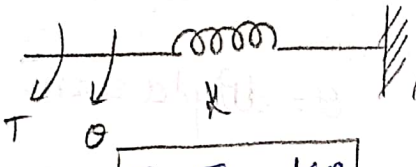
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ii) Dash Pot [B].

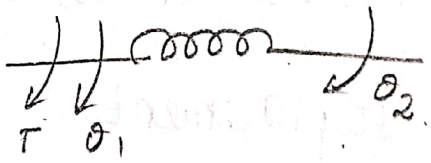


$$T = T_B = B\frac{d\theta}{dt}$$

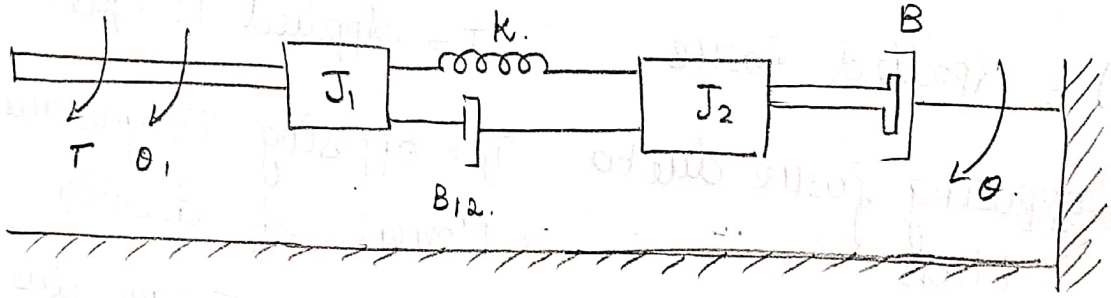
iii) Spring (K)



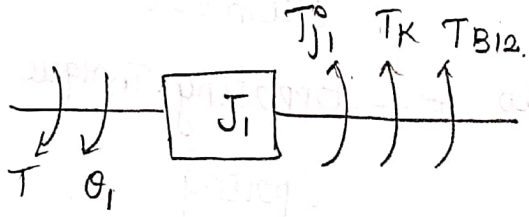
$$T = T_K = K\theta$$



1) Write the differential governing the Mechanical rotational System to determine $\frac{\theta(s)}{T(s)}$.



Node J_1:



Applied Torque = opposing Torque.

$$T = T_{J1} + T_K + T_{B12}.$$

$$T_{J1} = J_1\frac{d^2\theta_1}{dt^2}, \quad T_K = K(\theta_1 - \theta).$$

$$T_{B12} = B_{12}\frac{d}{dt}(\theta_1 - \theta).$$

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$$T = J_1 \frac{d^2\theta_1}{dt^2} + K(\theta_1 - \theta) + B_{12} \frac{d}{dt}(\theta_1 - \theta). \quad \text{— (1)}$$

Take Laplace Transform for above eqn.

$L.T\{T\} = T(S)$ $\qquad$ $L.T\left\{\frac{d^2}{dt^2}\right\} = S^2$

$L.T\{\theta\} = \theta(S)$ $\qquad$ $L.T\left\{\frac{d}{dt}\right\} = S.$

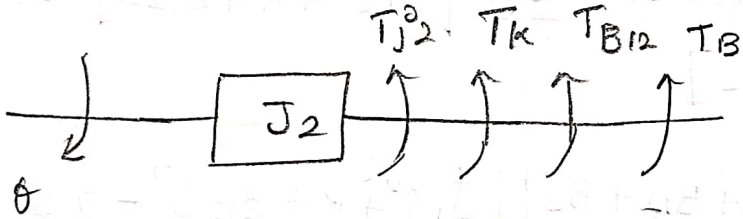
$L.T\{\theta_1\} = \theta_1(S)$

$$T(S) = J_1 S^2 \theta_1(S) + K[\theta_1(S) - \theta(S)] + B_{12} S[\theta_1(S) - \theta(S)]$$

$$T(S) = J_1 S^2 \theta_1(S) + K\theta_1(S) - K\theta(S) + B_{12} S\theta_1(S) - B_{12} S\theta(S).$$

$$T(S) = \theta_1(S)[J_1 S^2 + K + B_{12} S] - \theta(S)[K + B_{12} S] \quad \text{— (2)}$$

Node J_2 :-



Applied Torque = opposing Torque.

$$Q = T_{J_2} + T_K + T_{B12} + T_B.$$

$$T_{J_2} = J_2 \frac{d^2\theta}{dt^2} \qquad T_{B12} = B_{12} \frac{d}{dt}(\theta - \theta_1)$$

$$T_K = K(\theta_2 - \theta_1) \qquad T_B = B \frac{d\theta}{dt}.$$

$$Q = J_2 \frac{d^2\theta}{dt^2} + K(\theta - \theta_1) + B_{12} \frac{d}{dt}(\theta - \theta_1) + B \frac{d\theta}{dt}.$$

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Take Laplace Transform for above equation.

$$0 = J_2 S^2 \theta(S) + K[\theta(S) - \theta_1(S)] + B_{12} S[\theta(S) - \theta_1(S)] + B S \theta(S)$$

$$0 = J_2 S^2 \theta(S) + K\theta(S) - K\theta_1(S) + B_{12} S \theta(S) - B_{12} S \theta_1(S) + BS\theta(S)$$

$$0 = \theta(S)[J_2 S^2 + K + B_{12} + BS] - \theta_1(S)[K + B_{12} S].$$

$$\theta_1(S)[K + B_{12} S] = \theta(S)[J_2 S^2 + K + B_{12} + BS]$$

$$\theta_1(S) = \frac{\theta(S)[J_2 S^2 + K + B_{12} + BS]}{[K + B_{12} S]} \quad \text{— (3)}$$

Sub (3) in eqn (2).

$$T(S) = \frac{\theta(S)[J_2 S^2 + K + B_{12} + BS)}{[K + B_{12} S]} [J_1 S^2 + K + B_{12} S] - \theta(S)[K + B_{12} S]$$

$$T(S) = \frac{\theta(S)[J_2 S^2 + K + B_{12} + BS][J_1 S^2 + K + B_{12} S] - \theta(S)[K + B_{12} S]^2}{[K + B_{12} S]}$$

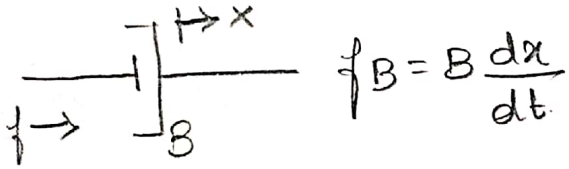
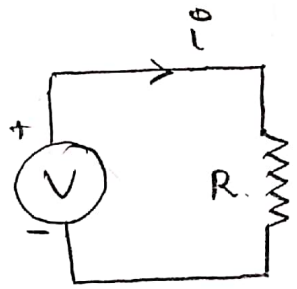
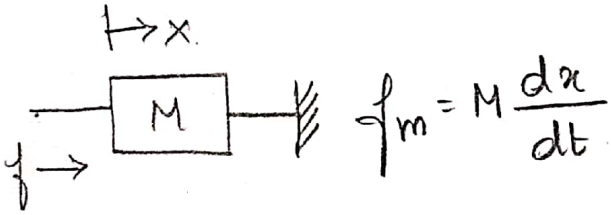
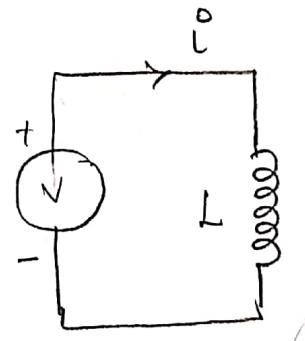
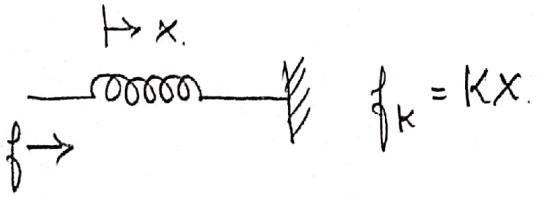
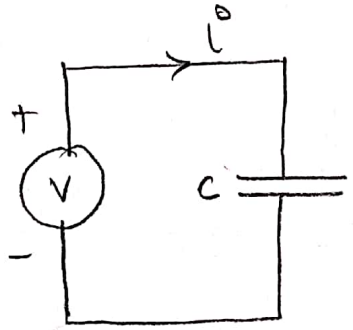
$$T(S) = \theta(S)\left\{\frac{[J_2 S^2 + K + B_{12} + BS][J_1 S^2 + K + B_{12} S] - [K + B_{12} S]^2}{K + B_{12} S}\right\}$$

$$\frac{\theta(S)}{T(S)} = \frac{K + B_{12} S}{[J_2 S^2 + K + B_{12} + BS][J_1 S^2 + K + B_{12} S] - [K + B_{12} S]^2}$$

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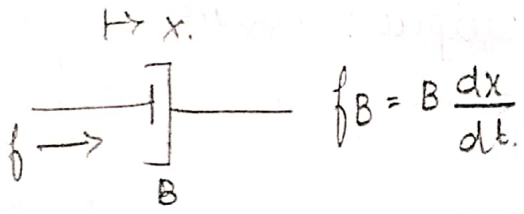
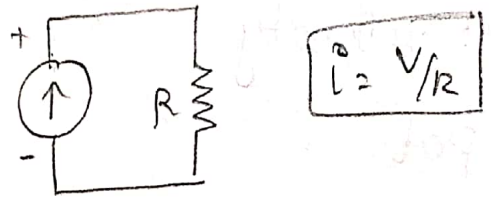
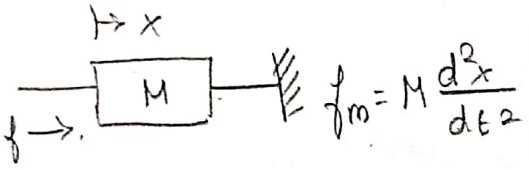
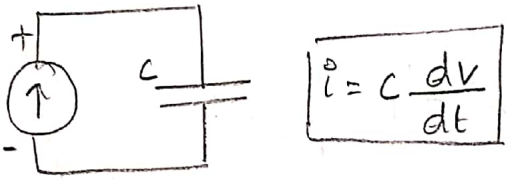
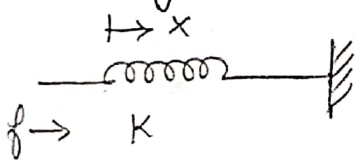
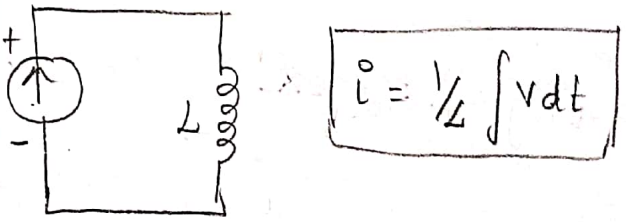
Electrical Analogy of Mechanical Translational System :

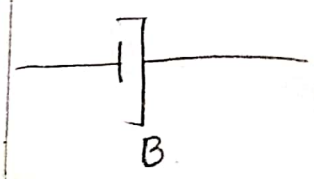
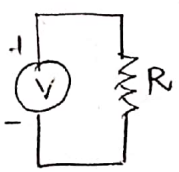
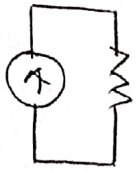
Force - Voltage. (KVL Mesh)

Mechanical System	Electrical System.
Input : Force output : velocity	Input : voltage output : current.
(i) Dash pot  $f_B = B\frac{dx}{dt}$	 $V = iR$
(ii) Mass [M] :-  $f_m = M\frac{dx}{dt}$	 $V = L\frac{di}{dt}$
(iii) Spring :-  $f_k = Kx$	 $V = \frac{1}{C}\int i\,dt$

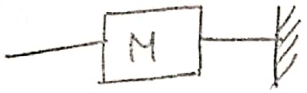
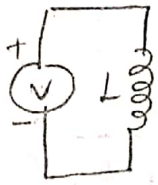
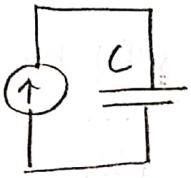
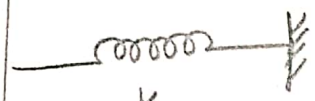
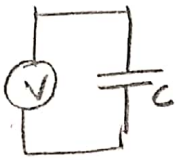
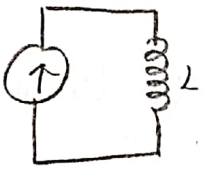
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Force-current (KCL Nodal).

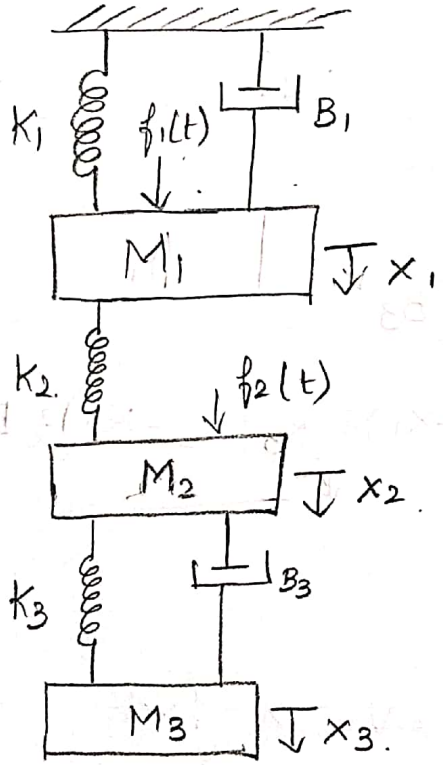
Mechanical System	Electrical System.
Input : force output : velocity	Input : current ouput : voltage.
(i) Dash Pot [B]:  	
(ii) Mass [M]:  	
(iii) Spring :-  $f_K = Kx$	

Element	Force voltage	Force - current.
 	 $V = iR$ $B \rightarrow R$	 $i = V/R$ $B \rightarrow 1/R$

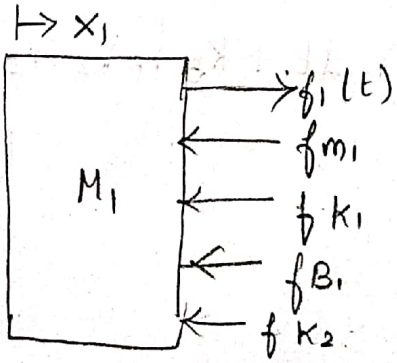
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		$V = L\frac{di}{dt}$ $M \rightarrow L$		$i = C\frac{dv}{dt}$ $M \rightarrow C$
		$V = \frac{1}{C}\int i\,dt$ $K \rightarrow \frac{1}{C}$		$i = \frac{1}{L}\int v\,dt$ $K \rightarrow \frac{1}{L}$

1) Write the differential Equation governing the Mechanical System shown in figure. Draw the force-voltage & force-current Electrical Analysis circuit and verify by Mesh and Node Equation.



Node M_1 :-



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$$f_1(t) = f_{m_1} + f_{K_1} + f_{B_1} + f_{K_2}$$

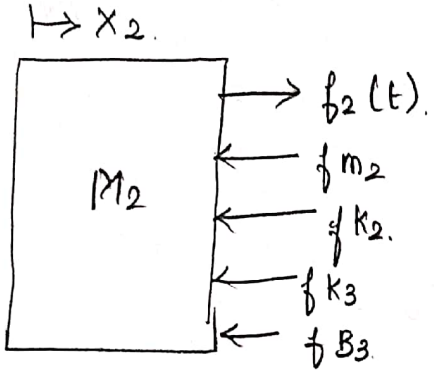
$$f_1(t) = M_1 \frac{d^2x_1}{dt^2} + K_1 x_1 + B_1 \frac{dx_1}{dt} + K_2 (x_1 - x_2)$$

Convert force to velocity.

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} \; ; \; \frac{dx}{dt} = v ; \quad x = \int v\,dt.$$

$$f_1(t) = M_1 \frac{dv_1}{dt} + K_1 \int v_1 dt + B_1 v_1 + K_2 \int (v_1 - v_2)\,dt \quad \text{—— (1)}$$

Node M_2 :



$$f_2(t) = f_{m_2} + f_{K_2} + f_{K_3} + f_{B_3}$$

$$f_2(t) = M_2 \frac{d^2x_2}{dt^2} + K_2 (x_2 - x_1) + K_3 (x_2 - x_3) + B_3 \frac{d}{dt}(x_2 - x_3)$$

Convert force to velocity

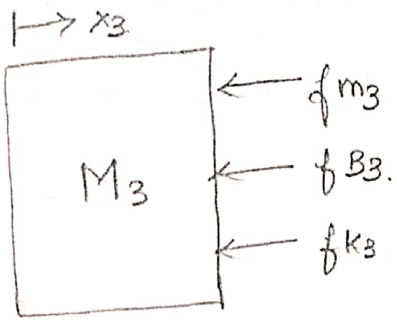
$$\frac{d^2x}{dt^2} = \frac{dv}{dt} \; ; \; \frac{dx}{dt} = v \; ; \; x = \int v dt.$$

$$f_2(t) = M_2 \frac{dv_2}{dt} + K_2 \int (v_2 - v_1)\,dt + K_3 \int (v_2 - v_3)\,dt + B_3 (v_2 - v_3)$$

—— (2)

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Node M_3 :



$$0 = f_{m3} + f_{B3} + f_{K3}.$$

$$0 = M_3 \frac{d^2x_3}{dt^2} + B_3 \frac{d}{dt}(x_3 - x_2) + K_3(x_3 - x_2)$$

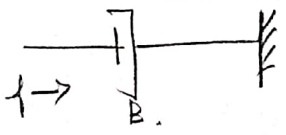
Convert displacement to velocity

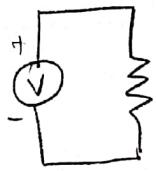
$$\frac{d^2x}{dt^2} = \frac{dv}{dt} \; ; \; \frac{dx}{dt} = v \; ; \; x = \int v\,dt.$$

$$0 = M_3 \frac{dv_3}{dt} + B_3(v_3 - x_2) + K_3 \int (v_3 - v_2)\,dt \quad \text{—— (3)}$$

Electrical Analogy of Force-Voltage.

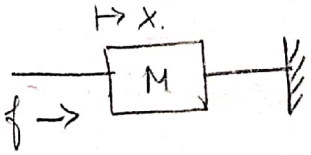
Mechanical	Electrical.
Input :- Force	Input : voltage v(t)
output : velocity	output : current i(t)

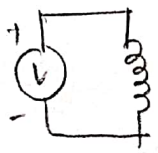




$V = iR$

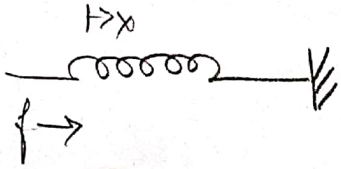
B → R

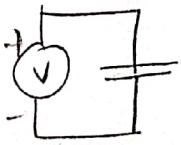




$V = L\,di/dt$

M → L





$V = 1/C \int i\,dt.$

K → 1/C

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$f_1(t) = V_1(t)$ | $M_1 \to L_1$ | $B_1 \to R_1$ | $K_1 \to 1/C_1$

$f_2(t) = V_2(t)$ | $M_2 \to L_2$ | $B_3 \to R_3$ | $K_2 \to 1/C_2$.

$V_1 \to \overset{\circ}{i}_1$ | $M_3 \to L_3$ | $K_3 \to 1/C_3$.

$V_2 \to \overset{\circ}{i}_2$

$V_3 \to \overset{\circ}{i}_3$.

Eqn ① $f_1(t) = M_1 \frac{dV_1}{dt} + K_1 \int V_1 dt + B_1 V_1 + K_2 \int (V_1 - V_2) dt$.

Mesh $\Rightarrow V_1(t) = L_1 \frac{d\overset{\circ}{i}_1}{dt} + \frac{1}{C_1} \int \overset{\circ}{i}_1 dt + R_1 \overset{\circ}{i}_1 + \frac{1}{C_2} \int (\overset{\circ}{i}_1 - \overset{\circ}{i}_2) dt$

Eqn ② $f_2(t) = M_2 \frac{dV_2}{dt} + K_2 \int (V_2 - V_1) dt + K_3 \int (V_2 - V_3) dt +$

$B_3 (V_2 - V_3)$

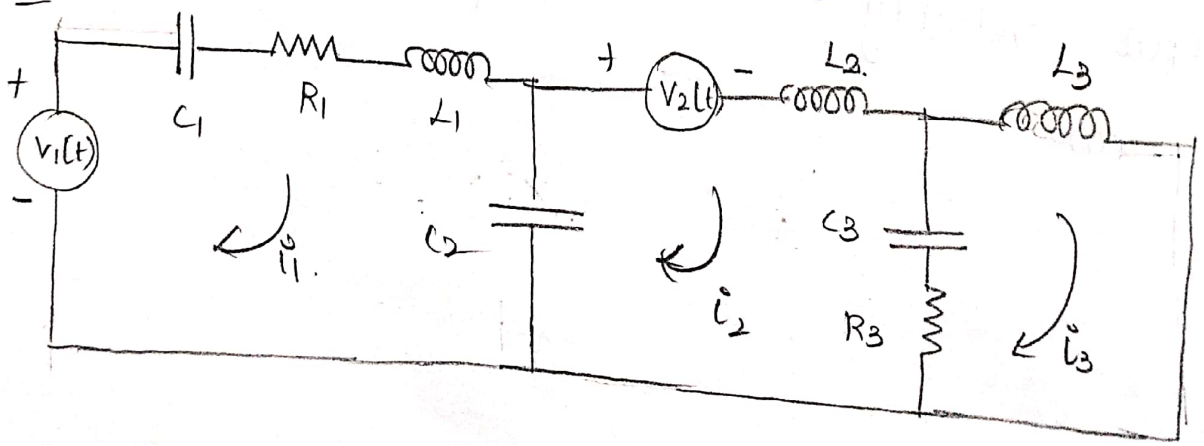
Mesh $\Rightarrow V_2(t) = L_2 \frac{d\overset{\circ}{i}_2}{dt} + \frac{1}{C_2} \int (\overset{\circ}{i}_2 - \overset{\circ}{i}_1) dt + \frac{1}{C_3} \int (\overset{\circ}{i}_2 - \overset{\circ}{i}_3) dt$

$+ R_3 (\overset{\circ}{i}_2 - \overset{\circ}{i}_3)$

Eqn ③ $0 = M_3 \frac{dV_3}{dt} + B_3 (V_3 - V_2) + K_3 \int (V_3 - V_2) dt$.

Mesh $\Rightarrow 0 = L_3 \frac{d\overset{\circ}{i}_3}{dt} + R_3 (\overset{\circ}{i}_3 - \overset{\circ}{i}_2) + \frac{1}{C_3} \int (\overset{\circ}{i}_3 - \overset{\circ}{i}_2) dt$.

Mesh



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Electrical Analogy of force - current.

$B \rightarrow 1/R$

$M \rightarrow C$

$K = 1/L$.

$f_1(t) = \dot{i}_1(t)$	$M_1 \rightarrow C_1$	$B_1 \rightarrow 1/R_1$	$K_1 \rightarrow 1/L_1$
$f_2(t) = \dot{i}_2(t)$	$M_2 \rightarrow C_2$	$B_3 \rightarrow 1/R_3$	$K_2 \rightarrow 1/L_2$
$V_1 \rightarrow V_1$	$M_3 \rightarrow C_3$		$K_3 \rightarrow 1/L_3$.
$V_2 \rightarrow V_2$			

Eqn ① ⇒ Nodal

$$\dot{i}_1(t) = C_1 \frac{dV_1}{dt} + \frac{1}{L}\int V_1\,dt + \frac{1}{R_1} V_1 + \frac{1}{L_2}\int (V_1 - V_2)\,dt.$$

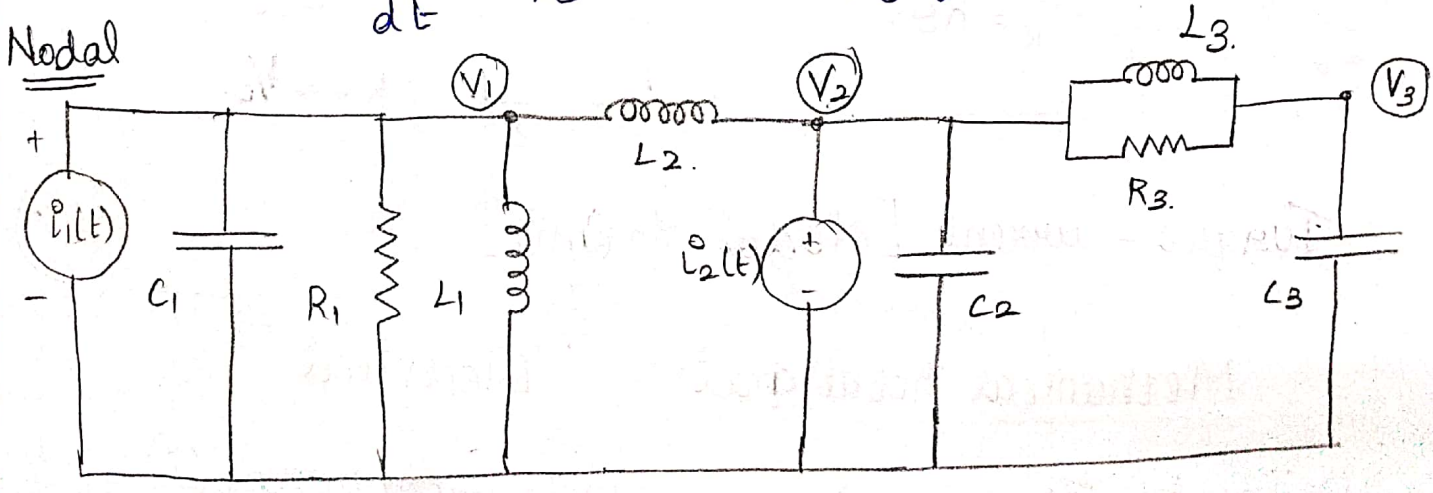
Eqn ② ⇒ Nodal

$$\dot{i}_2(t) = C_2 \frac{dV_2}{dt} + \frac{1}{L_2}\int (V_2 - V_1)\,dt + \frac{1}{L_3}\int (V_2 - V_3)\,dt + \frac{1}{R_3}(V_2 - V_3)$$

Eqn ③ ⇒ Nodal.

$$0 = C_3 \frac{dV_3}{dt} + \frac{1}{R_3}(V_3 - V_2) + \frac{1}{L_3}\int (V_3 - V_2)\,dt.$$

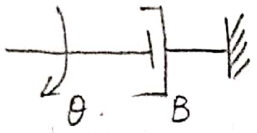
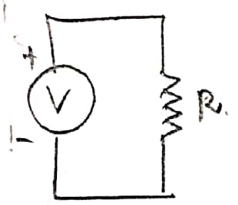
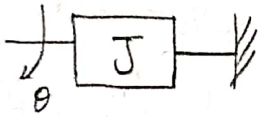
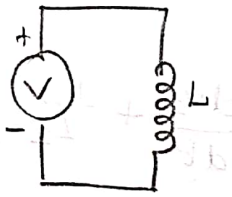
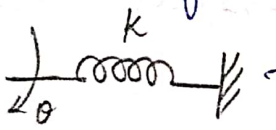
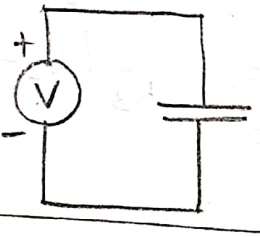
Nodal



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Electrical Analogy of Mechanical Rotational System

Torque - Voltage [Mesh Analysis]

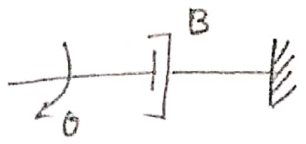
Mechanical Rotational	Electrical system.
(i) Input : Torque (T) output : Angular velocity (ω)	(i) Input : voltage (v) output : current (i)
(ii) Dash - Pot [B]:-  $T_B = B\frac{d\theta}{dt}$.	 $V = iR$. $B \rightarrow R$.
(iii) Moment of Interia [J]  $T_M = J\frac{d^2\theta}{dt^2}$	 $V = \frac{1}{L}\int i\,dt$. $M \rightarrow L$.
(iv) Spring [K]:  $T_K = K\theta$.	 $V = \frac{1}{C}\int i\,dt$. $K \rightarrow 1/C$.

Torque - current [Nodal Analysis]

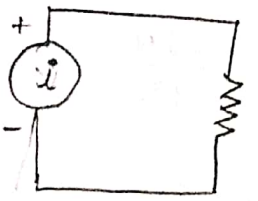
Mechanical Rotational	Electrical.
(i) Input : Torque (T) output : Angular velocity (ω)	(i) Input : current (i) output : voltage (v)

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(ii) Dash-pot [B].



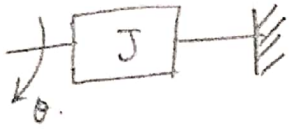
$T_B = B\frac{d\theta}{dt}$.



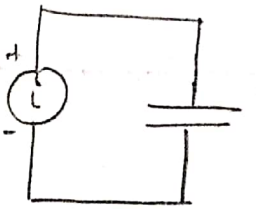
$I = \frac{V}{R}$

$B \rightarrow 1/R$.

(iii) Moment of Inertia [J]:



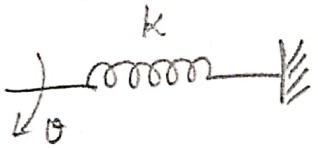
$T_j = J\frac{d^2\theta}{dt^2}$



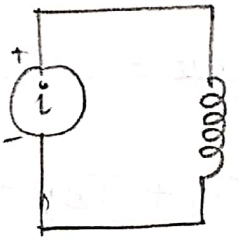
$I = C\frac{dv}{dt}$

$J \rightarrow C$.

(iv) Spring (K)



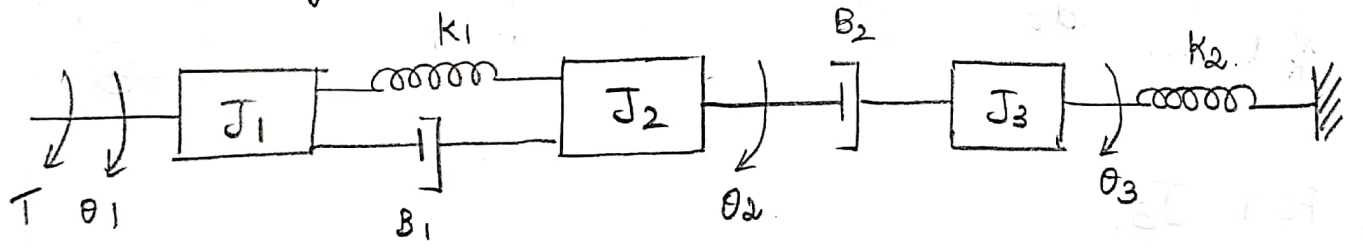
$T_K = K\theta$.



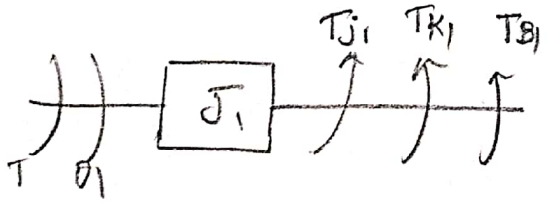
$I = \frac{1}{L}\int v\,dt$

$K = 1/L$.

Draw Torque- Voltage and Torque – current for Mechanical Rotational System.



(i) For J_1



Applied Torque = opposing Torque.

$$T = T_{j1} + T_{K1} + T_{B1}$$

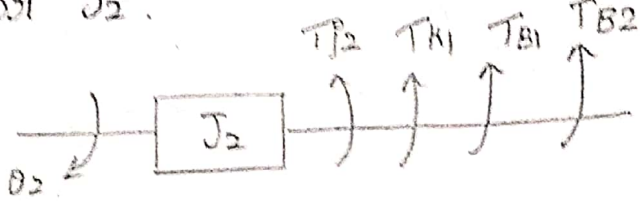
$$T = J_1\frac{d^2\theta_1}{dt^2} + K_1(\theta_1 - \theta_2) + B_1\frac{d}{dt}(\theta_1 - \theta_2)$$

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$$\left[\frac{d^2\theta}{dt^2} = \frac{d\omega}{dt};\ \frac{d\theta}{dt} = \omega;\ \theta = \int \omega\, dt\right]$$

$$T = J_1 \frac{d\omega_1}{dt} + K_1 \int (\omega_1 - \omega_2)\, dt + B_1(\omega_1 - \omega_2) \quad \text{— (1)}$$

(ii) For J_2.



Applied Torque = opposing Torque.

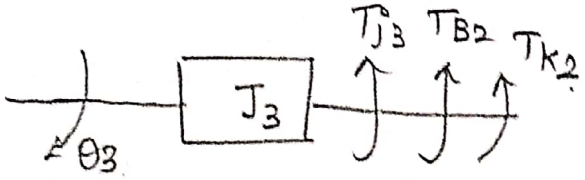
$$0 = T_{J2} + T_{K1} + T_{B1} + T_{B2}$$

$$0 = J_2 \frac{d^2\theta_2}{dt^2} + K_1(\theta_2 - \theta_1) + B_1 \frac{d}{dt}(\theta_2 - \theta_1) + B_2 \frac{d}{dt}(\theta_2 - \theta_3)$$

$$0 = J_2 \frac{d\omega_2}{dt} + K_1 \int (\omega_2 - \omega_1)\, dt + B_1(\omega_2 - \omega_1) + B_2(\omega_2 - \omega_3)$$

— (2)

(iii) For J_3.



$$0 = T_{J3} + T_{B2} + T_{K2}.$$

$$0 = J_3 \frac{d^2\theta}{dt^2} + B_2 \frac{d}{dt}(\theta_3 - \theta_2) + K_2 \theta_3.$$

$$0 = J_3 \frac{d\omega_3}{dt} + B_2(\omega_3 - \omega_2) + K_2 \int \omega_3\, dt \quad \text{— (3)}.$$

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Torque - Voltage (Mesh)

T-V(t) $\quad B_1 \to R_1 \quad J_1 \to L_1 \quad K_1 \to \frac{1}{C_1}$

$w_1 \to \dot{i}_1 \quad B_2 \to R_2 \quad J_2 \to L_2 \quad K_2 \to \frac{1}{C_2}$

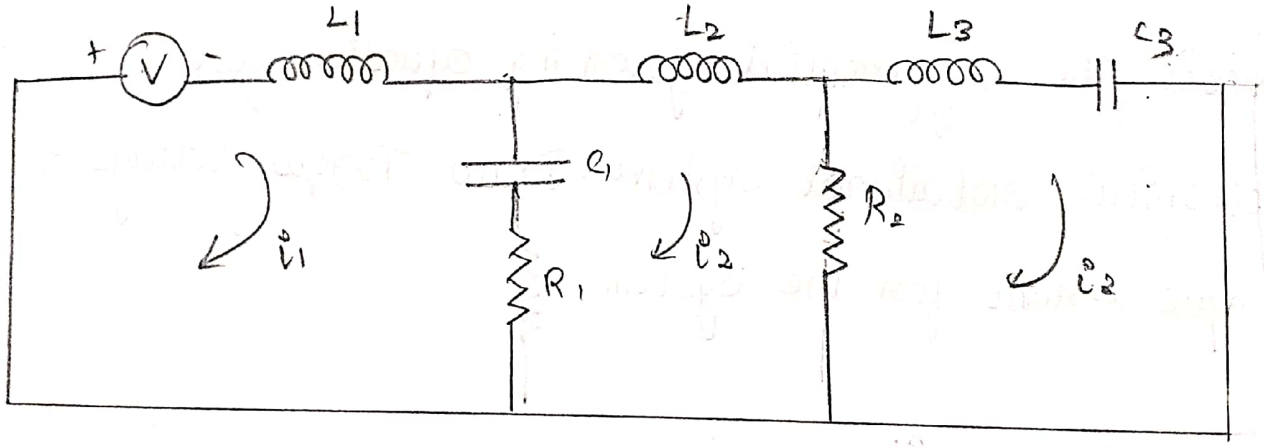
$w_2 \to \dot{i}_2 \quad J_3 \to L_3 \quad K_3 \to \frac{1}{C_3}$

Eqn ①, ② & ③ becomes.

$$V(t) = L_1 \frac{d\dot{i}_1}{dt} + \frac{1}{C_1}\int(\dot{i}_1 - \dot{i}_2)dt + R_1(\dot{i}_1 - \dot{i}_2)$$

$$0 = L_2 \frac{d\dot{i}_2}{dt} + \frac{1}{C_1}\int(\dot{i}_2 - \dot{i}_1)dt + R_1(\dot{i}_2 - \dot{i}_1) + R_2(\dot{i}_2 - \dot{i}_3)$$

$$0 = L_3 \frac{d\dot{i}_3}{dt} + R_2(\dot{i}_3 - \dot{i}_2) + \frac{1}{C_2}\int \dot{i}_3 dt.$$



Torque - current (Nodel).

I/P (T) $\quad$ I/P :- $\dot{i}$

O/P (ω) $\quad$ O/P :- V.

$T \to \dot{i}$

$w_1 \to V_1 \quad J_1 \to C_1 \quad B_1 \to 1/R_1 \quad K_1 \to 1/L_1$

$w_2 \to V_2 \quad J_2 \to C_2 \quad B_2 \to 1/R_2 \quad K_2 \to 1/L_2.$

$w_3 \to V_3 \quad J_3 \to C_3 \quad B_3 \to 1/R_3$

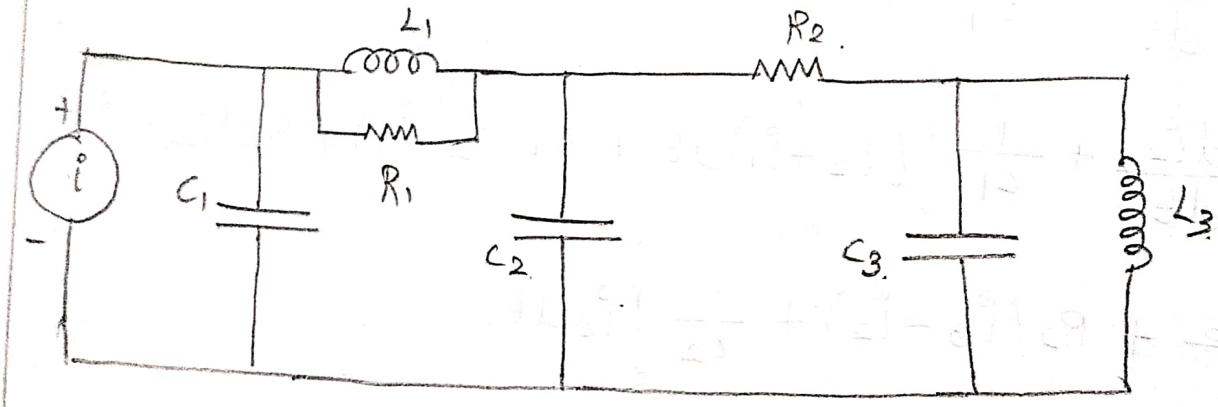
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eqn ①, ②, ③ becomes.

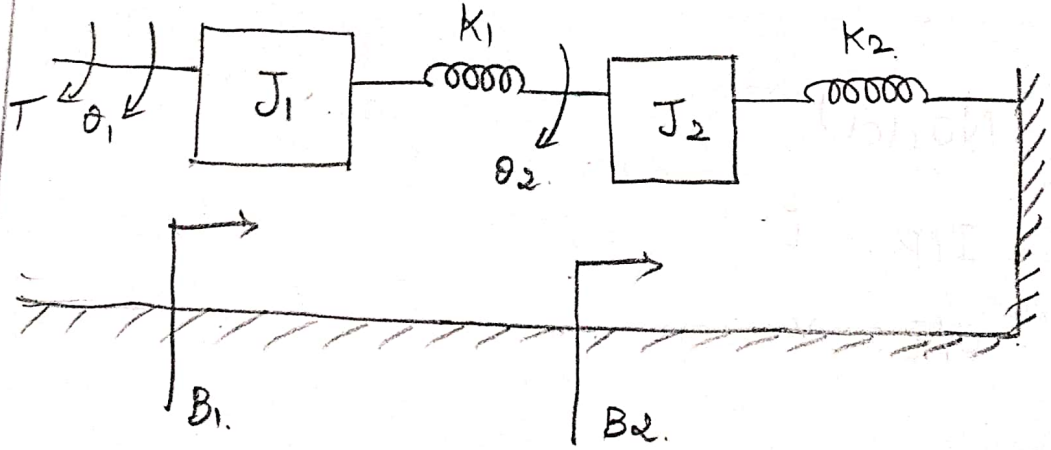
$$i = C_1 \frac{dV_1}{dt} + \frac{1}{L_1}\int (V_1 - V_2)\,dt + \frac{1}{R_1}(V_1 - V_2)$$

$$0 = C_2 \frac{dV_2}{dt} + \frac{1}{L_1}\int (V_2 - V_1)\,dt + \frac{1}{R_1}(V_2 - V_1) + \frac{1}{R_2}(V_2 - V_3)$$

$$0 = C_3 \frac{dV_3}{dt} + \frac{1}{R_2}(V_3 - V_2) + \frac{1}{L_2}\int V_3\,dt.$$



Write the differential governing equation for Mechanical rotational system. Draw Torque voltage & Torque current for the system.



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UNIT-II.

Time Response Analysis.

The Time Response of the System is the output of the closed Loop System as a function of time. It is denoted by c(t).

Closed Loop T.F $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = M(s)$.

The output Response is s-domain it can changed to c(t) whereas taking Laplace Inverse of C(s).

$$C(s) = R(s)\,M(s)$$

Response in time domain, $c(t) = L^{-1}\{C(s)\}$

$$C(t) = L^{-1}\{R(s)M(s)\}$$

Test signals:

Normally the input signals of the System are not known. Hence we use the test signals to know about the Input signals.

Standard test Signal.

1. a) Step signal.
   b) Unit step signal.

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2. a) Ramp Signal.

Unit Ramp Signal.

3. a) Parabolic Signal.

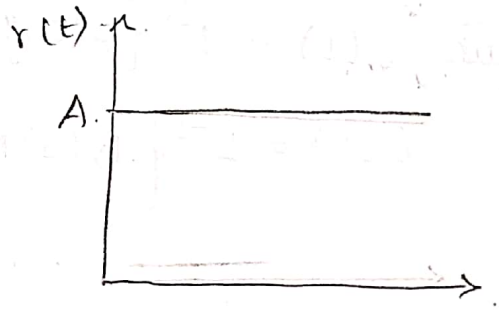
Unit parabolic Signal.

4. Impulse Signal.

5. Sinusoidal Signal.

1. Step Signal:

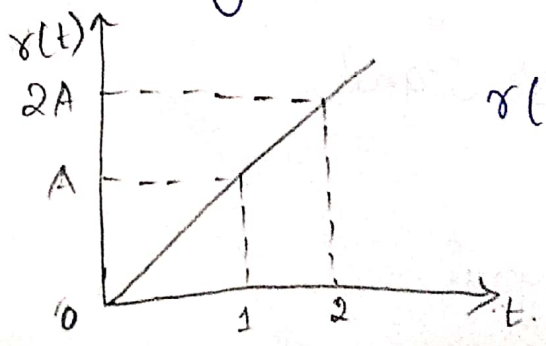
The Step Signal is a Signal whose value changes Zero to A.



$r(t) = A \; ; \; t \geq 0.$

2. Ramp Signal:-

The Ramp Signal is a Signal whose value Increases Linearly with time.

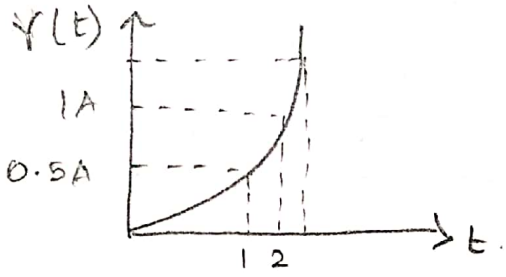


$r(t) = At.$

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3. Parabolic Signal :

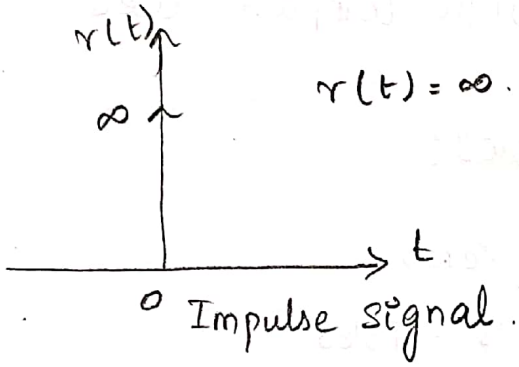
The value changes Square of the time.



$$r(t) = \frac{At^2}{2} \quad t \geq 0.$$

Impulse Signal :

A Signal vary Large Magnitude which is available short duration is called impulse signal.



Standard Test Signal.

Name of the signal	Time domain Signal r(t)	Laplace Transform Signal R(s)
Step	$r(t) = A$	$R(s) = A/s.$
Unit Step	$r(t) = 1$	$R(s) = 1/s.$
Ramp	$r(t) = At$	$R(s) = A/s^2$
Unit Ramp	$r(t) = t$	$R(s) = 1/s^2$

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Parabolic	$r(t) = \frac{At^2}{2}$	$R(s) = A/s^3$
Unit parabolic	$r(t) = \frac{t^2}{2}$	$R(s) = 1/s^3$.
Impluse	$r(t) = \infty$	$R(s) = 1$.

Review of Partial Fraction :-

Case(i) Function with Separate Poles.

Case(ii) Function with Multiple poles.

Case(iii) Function with complex poles.

Case (i) : Separate Poles :

$$T.F = \frac{K \rightarrow \text{Zeros.}}{s(s+p_1)(s+p_2) \rightarrow \text{Poles}}$$

For Ex :

$$T.F = \frac{2}{s(s+2)(s+4)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4}.$$

To find A, B, C.

$$A = T(s) \times s \Big|_{s=0}.$$

$$B = T(s) \times (s+2) \Big|_{s=-2}.$$

$$C = T(s) \times (s+4) \Big|_{s=-4}.$$

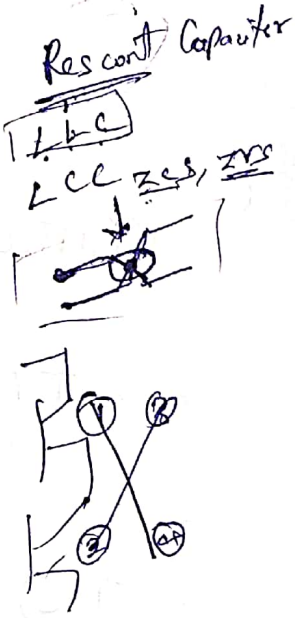
$$A = \frac{2}{s(s+2)(s+4)} \times s.$$

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$$= \frac{2}{(S+2)(S+4)}\Big|_{S=0}.$$

$$= \frac{2}{(0+2)(0+4)} = \frac{2}{2\times 4} = \frac{2}{8} = \frac{1}{4}.$$

$A = 0.25.$



$$B = T(S) \times (S+2)$$

$$= \frac{2}{S(S+2)(S+4)} \times (S+2)$$

$$= \frac{2}{S(S+4)}\Big|_{S=-2}.$$

$$= \frac{2}{-2(-2+4)} = \frac{2}{-2\times 2} = \frac{2}{-4} = -\frac{1}{2}.$$

$B = -0.5.$

$$C = T(S) \times (S+4)$$

$$= \frac{2}{S(S+2)(S+4)} \times (S+4).$$

$$= \frac{2}{S(S+2)}\Big|_{S=-4}$$

$$= \frac{2}{-4(-4+2)} = \frac{2}{-4(-2)} = \frac{2}{8} = \frac{1}{4}$$

$C = 0.25.$

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$$T(S) = \frac{2}{S(S+2)(S+4)}$$

$$= \frac{A}{S} + \frac{B}{S+2} + \frac{C}{S+4}.$$

$$T(S) = \frac{0.25}{S} - \frac{0.5}{S+2} + \frac{0.25}{S+4}.$$

Partial fraction with Multiple poles :

$$T(S) = \frac{K}{S(S+P_1)(S+P_2)^2}$$

$$T(S) = \frac{2}{S(S+1)(S+2)^2} = \frac{A}{S} + \frac{B}{S+1} + \frac{C}{(S+2)^2} + \frac{D}{(S+2)}$$

$$A = T(S) \times S \Big|_{S=0}.$$

$$B = T(S) \times (S+1) \Big|_{S=-1}.$$

$$C = T(S) \times (S+2)^2 \Big|_{S=-2}$$

$$D = \frac{d}{dS}\left[T(S) \times (S+2)^2\right]_{S=-2}.$$

$$A = T(S) \times S \Big|_{S=0}.$$

$$= \frac{2}{S(S+1)(S+2)} \times S \Big|_{S=0}.$$

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$$= \frac{2}{(0+1)(0+2)^2} = \frac{2}{4} = \frac{1}{2}$$

$A = 1/2$

$$B = T(s) \times (s+1)\Big|_{s=-1}$$

$$= \frac{2}{2(s+1)(s+2)^2} \times (s+1)\Big|_{s=-1}$$

$$= \frac{2}{-1(-1+2)^2} = \frac{2}{-1(1)^2}$$

$B = -2.$

$$C = T(s) \times (s+2)^2\Big|_{s=-2}$$

$$= \frac{2}{s(s+1)(s+2)^2} \times (s+2)^2\Big|_{s=-2.}$$

$$= \frac{2}{-2(-2+1)}$$

$$= \frac{2}{-2(-1)} = \frac{2}{2}.$$

$C = 1.$

$$D = \frac{d}{ds}\left[T(s) \times (s+2)^2\right]_{s=-2.}$$

$$= \frac{d}{ds}\left[\frac{2}{s(s+1)(s+2)^2} \times (s+2)^2\right]_{s=-2.}$$

$$= \frac{d}{ds}\left[\frac{2}{s(s+1)}\right]_{s=-2} = \frac{d}{ds}\left[\frac{2}{s^2+s}\right]_{s=2}$$ U/V.

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$$\frac{u}{v} = \frac{vu' - uv'}{v^2}$$

$$= \frac{s(s+1)(0) - 2(2s+1)}{(s^2+s^2)}\Bigg|_{s=-2}.$$

$$= \frac{0 - 2[2(-2)+1]}{[(-2)^2+(-2)^2]} = \frac{0-2[-4+1]}{[4-2]^2}.$$

$$D = \frac{-2(-3)}{(2)^2} = \frac{6}{4} = \frac{3}{2}.$$

$$D = 1.5.$$

$$T(s) = \frac{2}{s(s+1)(s+2)^2} = \frac{A}{s} + \frac{B}{(s+1)} + \frac{C}{(s+2)^2} + \frac{D}{s+2}$$

$$T(s) = \frac{0.5}{s} + \frac{2}{s+1} + \frac{1}{(s+2)^2} + \frac{1.5}{s+2}.$$

Partial Fraction complex poles:

$$T(s) = \frac{1}{(s+2)(s^2+s+1)} = \frac{A}{s+2} + \frac{Bs+C}{s^2+s+1}.$$

To find A,

$$A = T(s) \times (s+2)\Big|_{s=-2}.$$

$$= \frac{1}{(s+2)(s^2+s+1)} \times (s+2)\Big|_{s=-2}.$$

$$= \frac{1}{s^2+s+1}\Big|_{s=-2}.$$

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$$= \frac{1}{(-2)^2 - 2 + 1} = \frac{1}{4 - 2 + 1} = \frac{1}{3}$$

$$A = 0.33.$$

To find B & C.

$$\frac{1}{(s+2)(s^2+s+1)} = \frac{A}{s+2} + \frac{Bs+C}{s^2+s+1}$$

$$\frac{1}{(s+2)(s^2+s+1)} = \frac{A(s^2+s+1) + (Bs+C)(s+2)}{(s+2)(s^2+s+1)}$$

$$1 = A(s^2+s+1) + (Bs+C)(s+2).$$

We know $A = 0.33$.

$$1 = 0.33(s^2+s+1) + (Bs+C)(s+2)$$

$$1 = 0.33s^2 + 0.33s + 0.33 + Bs^2 + 2Bs + Cs + 2C.$$

$$1 = 0.33s^2 + 0.33s + 0.33 + Bs^2 + 2Bs + Cs + 2C.$$

Equate s^2.

$$0 = 0.33 + B.$$

$$B = -0.33.$$

Equate s.

$$0 = 0.33 + 2B + C.$$

$$0 = 0.33 + 2(-0.33) + C$$

$$-C = 0.33 - 0.66$$

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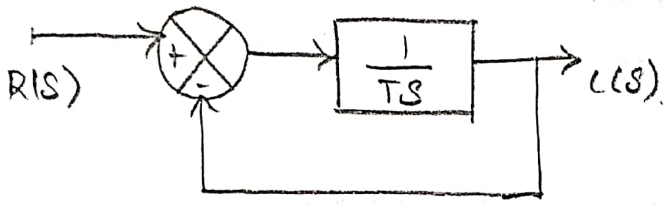
$-C = -0.33$.

$C = 0.33$.

$$T(S) = \frac{1}{(S+2)(S^2+S+1)} = \frac{A}{S+2} + \frac{BS+C}{S^2+S+1}$$

$$T(S) = \frac{0.33}{S+2} + \frac{(-0.33)S + 0.33}{S^2+S+1}$$

Response of first order system for unit step input :-



$G(S) = \frac{1}{TS}$.

$H(S) = 1$.

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1 + G(S)H(S)} = \frac{1/TS}{1 + \frac{1}{TS}(1)}$$

$$= \frac{1/TS}{\frac{TS+1}{TS}}$$

$$\frac{C(S)}{R(S)} = \frac{1}{1+TS}$$

$$C(S) = R(S) \cdot \frac{1}{1+TS}$$

Unit step input $R(S) = 1/S$.

$$C(S) = \frac{1}{S} \cdot \frac{1}{1+TS} = \frac{1}{S(1+TS)}$$

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$$C(s) = \frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$

$$A = C(s) \times s \Big|_{s=0}$$

$$= \frac{1}{s(1+Ts)} \times s \Big|_{s=0}$$

$$= \frac{1}{1+T(0)} = \frac{1}{1} = 1$$

$$A = 1.$$

$$B = C(s) \times (s+1) \Big|_{s=-1}$$

$$= \frac{1}{s(1+Ts)} \times (s+1) \Big|_{s=-1}$$

$$C(s) = \frac{1}{s(1+Ts)}$$

$$= \frac{k}{s(s+p_1)(s+p_2)} \rightarrow \text{Separate poles format.}$$

$$C(s) = \frac{1}{s(1+Ts)} = \frac{1}{sT\left(\frac{1}{T}+s\right)}$$

$$C(s) = \frac{1/T}{s(s+1/T)} = \frac{A}{s} + \frac{B}{s+1/T}$$

$$A = C(s) \times s \Big|_{s=0}$$

$$= \frac{1/T}{s(s+1/T)} \times s \Big|_{s=0}$$

$$A = \frac{1/T}{1/T} = 1$$

$$B = C(s) \times \left(s + \frac{1}{T}\right) \Big|_{s=-\frac{1}{T}}$$

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$$= \frac{1/T}{S(S+1/T)} \times (S+1/T)\Big|_{S=-1/T}$$

$$= \frac{1/T}{S}\Big|_{S=-1/T}.$$

$$= \frac{1/T}{-1/T}$$

$B = -1.$

$$C(S) = \frac{A}{S} + \frac{B}{S+1/T}$$

$$= \frac{1}{S} - \frac{1}{S+1/T}$$

To find $c(t)$:-

$$c(t) = L^{-1}\{C(S)\}$$

$$= L^{-1}\left\{\frac{1}{S} - \frac{1}{S+1/T}\right\}$$

$$c(t) = 1 - e^{-t/T}$$

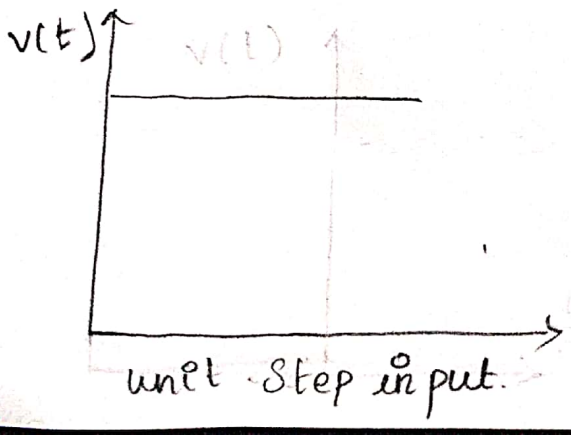
$\left[\because L^{-1}\left\{\frac{1}{S}\right\} = 1 \quad L^{-1}\left\{\frac{1}{S+a}\right\} = e^{-at}\right]$

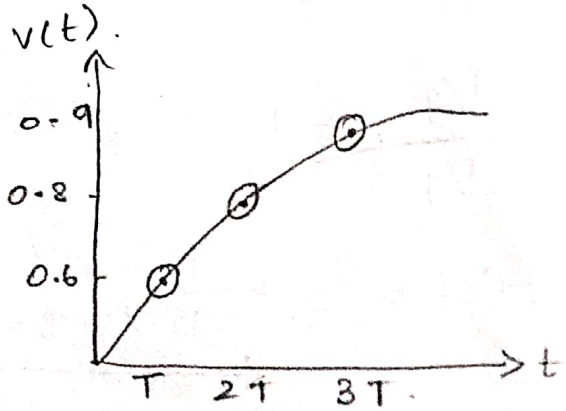
Where $t=0$; $c(t) = 1 - e^{0} = 0$

$t = 1T$; $c(t) = 1 - e^{-1} = +0.632.$

$t = 2T$; $c(t) = 1 - e^{-2} = 0.865.$

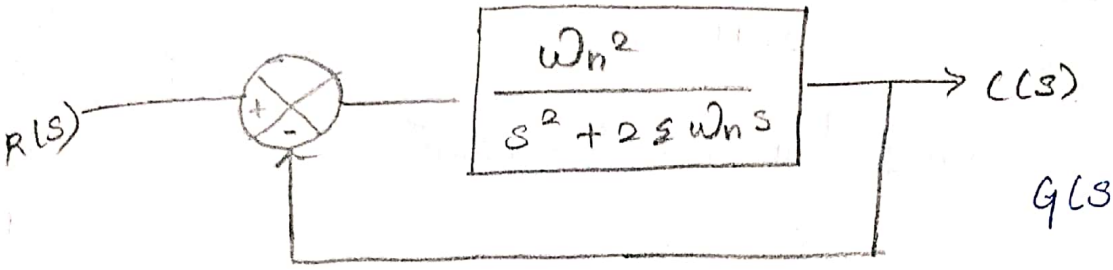
$t = 3T$; $c(t) = 1 - e^{-3} = 0.95.$





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Second order System :-



$$G(S) = \frac{\omega_n^2}{s^2+2\xi\omega_n s}$$

$$H(S) = 1.$$

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)H(S)}$$

$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$\xi \rightarrow$ Damping Ratio.

$\omega_n \rightarrow$ Undamped natural frequency.

Depend upon ξ the s/m can be classified into 4 class :-

case (i) : $\xi = 0$ (Undamped System).

case (ii) : $0 < \xi < 1$ (Underdamped System).

case (iii) : $\xi = 1$ (Critically damped System).

case (iv) : $\xi > 1$ (overdamped System).

The characteristic Equation of 2nd order System is

$$s^2 + 2\xi\omega_n + \omega_n^2 = 0.$$

$$a = 1, \quad b = +2\xi\omega_n, \quad c = \omega_n^2.$$

Sub the value is quadratic Equation.

$$s_1, s_2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}.$$

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$$= \frac{-2\xi\omega_n \pm \sqrt{(2\xi\omega_n)^2 - 4(1)\omega_n^2}}{2(1)}$$

$$= \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$= \frac{-2\xi\omega_n \pm \sqrt{4\omega_n^2(\xi^2-1)}}{2}$$

$$= \frac{-2\xi\omega_n \pm 2\omega_n\sqrt{\xi^2-1}}{2}$$

$$= \frac{2(-\xi\omega_n \pm \omega_n\sqrt{\xi^2-1})}{2}$$

$$S_1 S_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1} \quad \text{—①}$$

In Eqn ①.

$\xi = 0$; $S_1, S_2 = \pm j\omega_n$ (Roots are purely imaginary) System is Undamped)

$\xi = 1$; $S_1, S_2 = -\omega_n$ (Roots are Real the System is Critically damped).

$\xi > 1$; $S_1, S_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$ (Roots are Real and Unequal System is Overdamped)

$0 < \xi < 1$; $S_1, S_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi_1^2-1}$

$$= -\xi\omega_n \pm \omega_n\sqrt{(-1)(1-\xi^2)}$$

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$= -\xi\omega_n \pm \omega_n\sqrt{-1}\sqrt{1-\xi^2}$ $\quad [\because \sqrt{-1} = j$

$= -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$ $\quad \omega_n\sqrt{1-\xi^2} = \omega_d^2]$

$= -\xi\omega_n \pm j\omega_d$ [Roots are complex conjugate system is underdamped].

Response of the second order system undamped for unit step Input :-

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$\xi = 0.$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + \omega_n^2}$$

$$C(s) = R(s) \cdot \frac{\omega_n^2}{s^2 + \omega_n^2}$$

$$C(s) = \frac{1}{s} \cdot \frac{\omega_n^2}{s^2 + \omega_n^2}.$$

$$C(s) = \frac{\omega_n^2}{s(s^2 + \omega_n^2)} = \frac{A}{s} + \frac{B}{s^2 + \omega_n^2}$$

$$A = C(s) \times s\Big|_{s=0}.$$

$$= \frac{\omega_n^2}{s(s + \omega_n^2)} \times s\Big|_{s=0}.$$

$$= \frac{\omega_n^2}{0 + \omega_n^2} = \frac{\omega_n^2}{\omega_n^2}$$

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$A = 1.$

$$B = C(s) \times (s^2 + \omega_n^2) \Big|_{s^2 = -\omega_n^2}$$

$$s^2 = -\omega_n^2$$

$$\sqrt{s^2} = \sqrt{-\omega_n^2}$$

$$s = \pm j\omega_n$$

$$B = C(s) \times (s^2 + \omega_n^2) \Big|_{s = +j\omega_n}$$

$$= \frac{\omega_n^2}{s(s^2 + \omega_n^2)} \times (s^2 + \omega_n^2)$$

$$= \frac{\omega_n^2}{+j\omega_n} \times \frac{j}{j} \quad -j\omega_n = -s.$$

$$= \frac{j\omega_n}{j^2} = -j\omega_n = -s \qquad \boxed{B = -s}$$

$$\frac{\omega_n^2}{s} \Big|_{s = j\omega_n}$$

$$= \frac{\omega_n^2}{j\omega_n}$$

$$B = -s. \; \frac{\omega_n}{j}$$

$$C(s) = \frac{A}{s} + \frac{B}{s^2 + \omega_n^2}$$

$$C(s) = \frac{1}{s} - \frac{s}{s^2 + \omega_n^2}$$

$$C(t) = L^{-1}\{C(s)\}$$

$$= L^{-1}\left\{\frac{1}{s} - \frac{s}{s^2 + \omega_n^2}\right\}$$

$$L^{-1}\left\{\frac{1}{s}\right\} = 1 \qquad L^{-1}\left\{\frac{s}{s^2 + \omega^2}\right\} = \cos \omega t.$$

$$C(t) = L^{-1}\left\{\frac{1}{s} - \frac{s}{s^2 + \omega_n^2}\right\}$$

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$c(t) = 1 - \cos \omega_n t$.

Response of Second order System critically damped for unit step Input:

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\zeta = 1$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2}$$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{(s+\omega_n)^2}$$

$$C(s) = R(s) \frac{\omega_n^2}{(s+\omega_n)^2}$$

$$C(s) = \frac{1}{s} \cdot \frac{\omega_n^2}{(s+\omega_n)^2}$$

$$C(s) = \frac{\omega_n^2}{s(s+\omega_n)^2} = \frac{A}{s} + \frac{B}{(s+\omega_n)^2} + \frac{C}{(s+\omega_n)}.$$

$$A = C(s) \times s \Big|_{s=0}$$

$$= \frac{\omega_n^2}{s(s+\omega_n)^2} \times s \Big|_{s=0}$$

$$= \frac{\omega_n^2}{0 + \omega_n^2}$$

$$A = 1$$

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$$B = C(s) \times (s+\omega_n)^2 \Big|_{s=-\omega_n}.$$

$$= \frac{\omega_n^2}{s(s+\omega_n)^2} \times (s+\omega_n)^2 \Big|_{s=-\omega_n}.$$

$$= \frac{\omega_n^2}{-\omega_n}.$$

$$\boxed{B = -\omega_n}$$

$$C = \frac{d}{ds}\left[C(s) \times (s+\omega_n)^2\right]_{s=-\omega_n}.$$

$$= \frac{d}{ds}\left[\frac{\omega_n^2}{s(s+\omega_n)^2} \times (s+\omega_n)^2\right]_{s=-\omega_n}$$

$$= \frac{d}{ds}\left[\frac{\omega_n^2}{s}\right]\Big|_{s=-\omega_n}$$

$$\frac{u}{v} = \frac{vu' - uv'}{v^2}.$$

$$= \frac{s(0) - \omega_n^2(1)}{s^2}$$

$$= \frac{-\omega_n^2}{s^2}\Big|_{s=-\omega_n}$$

$$= \frac{-\omega_n^2}{(-\omega_n)^2} = -1$$

$$C = -1$$

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$$C(s) = \frac{A}{s} + \frac{B}{(s+\omega_n)^2} + \frac{C}{s+\omega_n}$$

$$C(s) = \frac{1}{s} - \frac{\omega_n}{(s+\omega_n)^2} - \frac{1}{s+\omega_n}$$

$$c(t) = L^{-1}\{C(s)\}$$

$$= L^{-1}\left\{\frac{1}{s} - \frac{\omega_n}{(s+\omega_n)^2} - \frac{1}{s+\omega_n}\right\}$$

Response of 2nd order system for over damped for unit step input :

$\xi > 1$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$s_a, s_b = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

$$= -\left[\xi\omega_n + \omega_n\sqrt{\xi^2 - 1}\right]$$

$$s_1 = \xi\omega_n - \omega_n\sqrt{\xi^2 - 1}$$

$$s_2 = \xi\omega_n + \omega_n\sqrt{\xi^2 - 1}$$

$$s_1 = -s_a \qquad s_2 = -s_b.$$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{\omega_n^2}{(s+s_1)(s+s_2)}$$

$$C(s) = R(s)\cdot\frac{\omega_n^2}{(s+s_1)(s+s_2)}$$

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Unit step $R(s) = \frac{1}{s}$

$$C(s) = \frac{1}{s} \frac{\omega_n^2}{(s+s_1)(s+s_2)}$$

$$C(s) = \frac{\omega_n^2}{s(s+s_1)(s+s_2)} = \frac{A}{s} + \frac{B}{(s+s_1)} + \frac{C}{(s+s_2)}$$

$$A = C(s) \times s \Big|_{s=0}$$

$$= \frac{\omega_n^2}{s(s+s_1)(s+s_2)} \times s \Big|_{s=0}$$

$$= \frac{\omega_n^2}{(0+s_1)(0+s_2)} = \frac{\omega_n^2}{s_1 s_2}$$

$$A = \frac{\omega_n^2}{\left[\zeta\omega_n - \omega_n\sqrt{\zeta^2-1}\right]\left[\zeta\omega_n + \omega_n\sqrt{\zeta^2-1}\right]}$$

$$A = \frac{\omega_n^2}{\zeta^2\omega_n^2 - \omega_n^2(\zeta^2-1)}$$

$$= \frac{\omega_n^2}{\zeta^2\omega_n^2 - \zeta^2\omega_n^2 + \omega_n^2}$$

$$A = \frac{\omega_n^2}{\omega_n^2}$$

$$A = 1$$

$$B = C(s) \times (s+s_1) \Big|_{s=-s_1}$$

$$= \frac{\omega_n^2 \ (s+s_1)}{s(s+s_1)(s+s_2)} \Big|_{s=-s_1}$$

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$$= \frac{\omega_n^2}{s(s+s_2)}\Big|_{s=-s_2} = \frac{\omega_n^2}{-s_1(-s_1+s_2)}$$

$$= \frac{\omega_n^2}{-s_1\left[-\zeta\omega_n - \omega_n\sqrt{\zeta^2-1} + \zeta\omega_n + \omega_n\sqrt{\zeta^2-1}\right]}$$

$$= \frac{\omega_n^2}{-s_1\left(2\omega_n\sqrt{\zeta^2-1}\right)}$$

$$B = \frac{-\omega_n}{s_1 2\sqrt{\zeta^2-1}} = \frac{-\omega_n}{2\sqrt{\zeta^2-1}} \cdot \frac{1}{s_1}$$

$$C = C(s) \times (s+s_2)\Big|_{s=-s_2}$$

$$= \frac{\omega_n^2}{s(s+s_1)(s+s_2)}(s+s_2)\Big|_{s=-s_2}.$$

$$= \frac{\omega_n^2}{s(s+s_1)}\Big|_{s=-s_2}$$

$$= \frac{\omega_n^2}{-s_2(-s_2+s_1)}$$

$$s_1 = \zeta\omega_n - \omega_n\sqrt{\zeta^2-1}$$

$$s_2 = \zeta\omega_n + \omega_n\sqrt{\zeta^2-1}$$

$$= \frac{\omega_n^2}{-s_2\left[-\zeta\omega_n - \omega_n\sqrt{\zeta^2-1} + \zeta\omega_n - \omega_n\sqrt{\zeta^2-1}\right]}$$

$$= \frac{\omega_n^2}{-s_2\left[-2\omega_n\sqrt{\zeta^2-1}\right]}$$

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$$= \frac{\omega_n^2}{s_2\left[2\omega_n\sqrt{\xi^2-1}\right]}$$

$$c = \frac{\omega_n}{s_2 2\sqrt{\xi^2-1}} = \frac{\omega_n}{2\sqrt{\xi^2-1}} \cdot \frac{1}{s_2}.$$

$$C(s) = \frac{A}{s} + \frac{B}{s+s_1} + \frac{C}{s+s_2}$$

$$C(s) = \frac{1}{s} - \frac{\omega_n}{2\sqrt{\xi^2-1}} \cdot \frac{1}{s_1} \cdot \frac{1}{s+s_1} + \frac{\omega_n}{2\sqrt{\xi^2-1}} \cdot \frac{1}{s_2} \cdot \frac{1}{s+s_2}$$

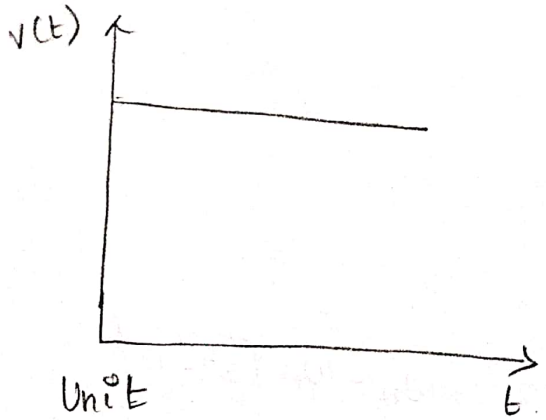
$$c(t) = L^{-1}\{C(s)\}$$

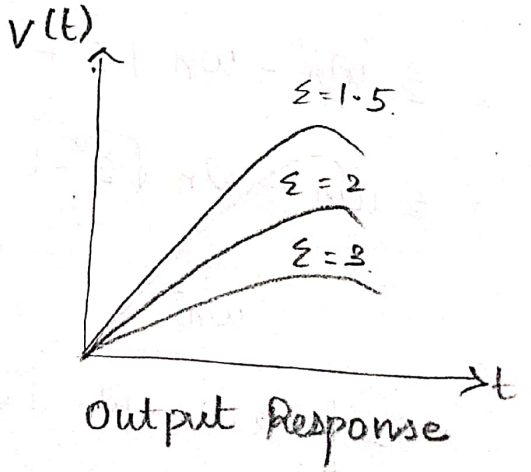
$$= L^{-1}\left\{\frac{1}{s} - \frac{\omega_n}{2\sqrt{\xi^2-1}} \cdot \frac{1}{s_1} \cdot \frac{1}{s+s_1} + \frac{\omega_n}{2\sqrt{\xi^2-1}} \cdot \frac{1}{s_2} \cdot \frac{1}{s+s_2}\right.$$

$$L^{-1}\{1/s\} = 1$$

$$L^{-1}\left\{\frac{1}{s+a}\right\} = e^{-at}.$$

$$c(t) = \left[1 - \frac{\omega_n}{2\sqrt{\xi^2-1}} \cdot \frac{1}{s_1} e^{-s_1 t} + \frac{\omega_n}{2\sqrt{\xi^2-1}} \cdot \frac{1}{s_2} e^{-s_2 t}\right]$$





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Response of overdamped System for 2nd order unit step Input :-

$$0 < \xi < 1$$

$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2} \qquad \omega_d = \omega_n\sqrt{1-\xi^2}$$

$$C(S) = R(S)\frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2}$$

$$C(S) = \frac{\omega_n^2}{S(S^2 + 2\xi\omega_n S + \omega_n^2)}$$

$$C(S) = \frac{\omega_n^2}{S(S^2 + 2\xi\omega_n S + \omega_n^2)} = \frac{A}{S} + \frac{BS + C}{BS^2 + 2\xi\omega_n S + \omega_n^2}$$

$$A = C(S) \times S\big|_{S=0}$$

$$= \frac{\omega_n^2}{S(S^2 + 2\xi\omega_n S + \omega_n^2)} \times S\Big|_{S=0} = \frac{\omega_n^2}{\omega_n^2}$$

$$A = 1.$$

To find B & C.

$$\frac{\omega_n^2}{S(S^2 + 2\xi\omega_n S + \omega_n^2)} = \frac{A}{S} + \frac{BS + C}{S^2 + 2\xi\omega_n + \omega_n^2}$$

$$\frac{\omega_n^2}{S(S^2 + 2\xi\omega_n S + \omega_n^2)} = \frac{A(S^2 + 2\xi\omega_n S + \omega_n^2) + (BS + C)(S)}{S(S^2 + 2\xi\omega_n S + \omega_n^2)}$$

$$\omega_n^2 = S^2 + 2\xi\omega_n S + \omega_n^2 + BS^2 + CS.$$

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Equate s^2

$0 = 1 + B$.

$B = -1$.

Equate s

$0 = 2\xi\omega_n + C$.

$C = -2\xi\omega_n$

$$C(s) = \frac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)} = \frac{1}{s} + \frac{-s-2\xi\omega_n}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$\left[L^{-1}\left\{\frac{1}{s}\right\} = 1 \; ; \; L^{-1}\left\{\frac{\omega}{(s+a)^2+\omega^2}\right\} = e^{-at}\sin\omega t \right.$$

$$\left. L^{-1}\left\{\frac{s+a}{(s+a)^2+\omega^2}\right\} = e^{-at}\cos\omega t \right]$$

$$C(t) = L^{-1}\{C(s)\}$$

+ + − by $\xi^2\omega_n^2$ in denominator.

$$C(s) = \frac{1}{s} - \frac{(s+2\xi\omega_n)}{s^2+2\xi\omega_n s+\omega_n^2+\xi^2\omega_n^2-\xi^2\omega_n^2}$$

$$= \frac{1}{s} - \frac{(s+2\xi\omega_n)}{(s^2+2\xi\omega_n s+\xi^2\omega_n^2)+\omega_n^2-\xi^2\omega_n^2}$$

$$= \frac{1}{s} - \frac{(s+2\xi\omega_n)}{(s+\xi\omega_n)^2+\omega_n^2-\xi^2\omega_n^2}$$

$$= \frac{1}{s} - \frac{(s+2\xi\omega_n)}{(s+\xi\omega_n)^2+\omega_n^2(1-\xi^2)} \qquad \left[\because \omega_d = \omega_n\sqrt{1-\xi^2}\right]$$

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$$= \frac{1}{s} - \frac{(s+\xi\omega_n)}{(s+\xi\omega_n)^2+\omega_d^2} - \frac{\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2}$$

$$C(s) = \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} - \frac{\xi\omega_n}{\omega_d}\cdot\frac{\omega_d}{(s+\xi\omega_n)^2+\omega_d^2}$$

$$c(t) = L^{-1}C(s)$$

$$c(t) = L^{-1}\left\{\frac{1}{s} - \frac{(s+\xi\omega_n)}{(s+\xi\omega_n)^2+\omega_d^2} - \frac{\xi\omega_n}{\omega_d}\cdot\frac{\omega_d}{(s+\xi\omega_n)^2+\omega_d^2}\right\}$$

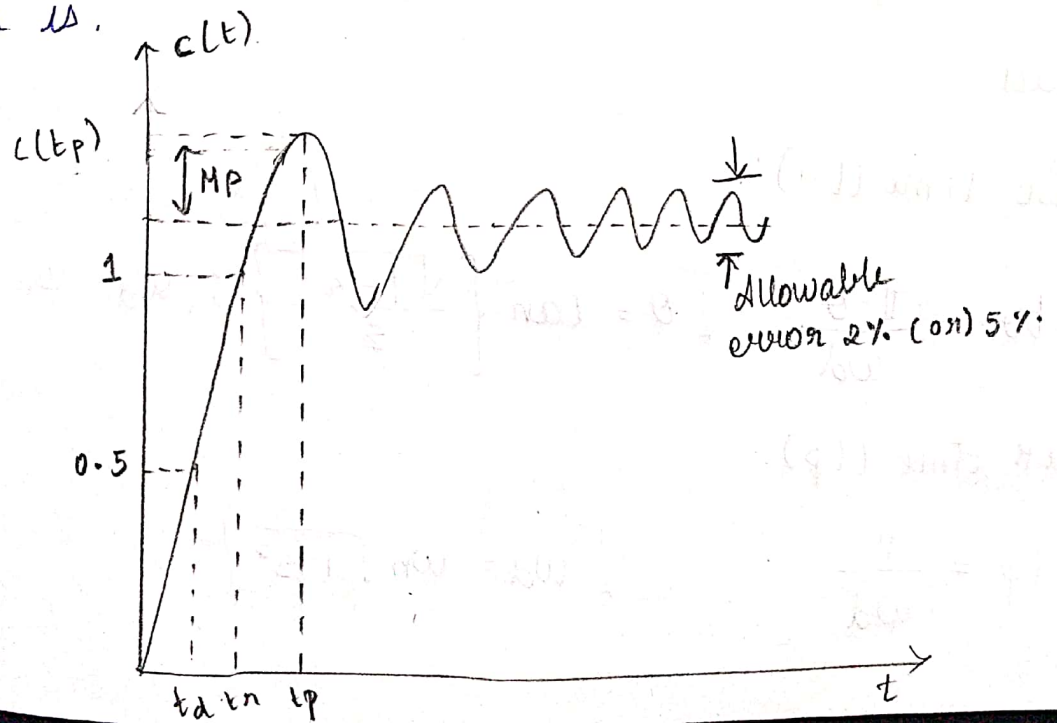
$$c(t) = 1 - e^{-\xi\omega_n t}\sin\omega_d t - \frac{\xi\omega_n}{\omega_d}e^{-\xi\omega_n t}\cos\omega_d t.$$

$$c(t) = 1 - e^{-\xi\omega_n t}\left(\sin\omega_d t - \frac{\xi\omega_n}{\omega_d}\cos\omega_d t\right)$$

Time Domain Specifications :-

The transient Response of control system often Exibits damped oscillation before Reaching steady state.

The typical damped oscillatory Response of the System is.



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1. Delay time (td)

It is taken for Response to Reach 50% of final value for short time.

2. Rise time: (tr)

(0 to 100% at very time)

3. Peak time (tp):-

To Reach Peak value for the first time.

4. Peak overshoot (%. Mp)

It is defined as Maximum Peak value to the final Peak value.

5. Setting time (ts)

It is defined as time taken to Response to stay within specified error.

Formula:-

1. Rise time (tr):

$$t_r = \frac{\pi - \theta}{\omega_d} \qquad \left[\theta = \tan^{-1}\left\{\frac{\sqrt{1-\xi^2}}{\xi}\right\};\ \omega_d = \omega_n\sqrt{1-\xi^2}\right]$$

2. Peak time (tp):-

$$t_p = \frac{\pi}{\omega_d} \qquad \left[\omega_d = \omega_n\sqrt{1-\xi^2}\right]$$

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3) Peak Overshoot (% Mp):-

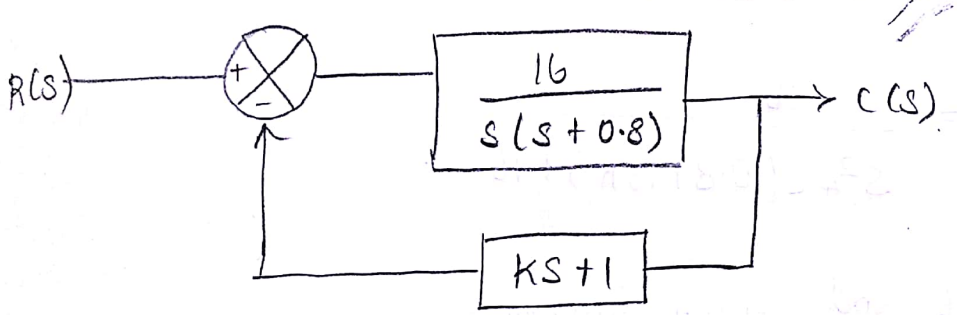
$$\% M_p = \left\{ e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} \right\} \times 100.$$

4. Settling time (t_s)

$t_s = 4T$ (for 2% error) $\quad \left[T = \frac{1}{\xi \omega_n}\right]$

$t_s = 5T$ (for 5% error)

A positional control System with velocity feedback shown in figure. what is the time of c(t) to the Unit Step Input Given that $\xi = 0.5$. Also calculate Rise, time, peak time, Maximum Overshoot and settling time.



To find

$t_p = ?$ $\quad C(t) = ?$

$t_r = ?$ $\quad \%.Mp = ?$

$t_s = ?$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

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$$G(s) = \frac{16}{s(s+0.8)}$$

$$H(s) = Ks + 1$$

$$\frac{C(s)}{R(s)} = \frac{\frac{16}{s(s+0.8)}}{1 + \frac{16}{s(s+0.8)}(Ks+1)}$$

$$\frac{C(s)}{R(s)} = \frac{\frac{16}{s(s+0.8)}}{\frac{s(s+0.8) + 16(Ks+1)}{s(s+0.8)}}$$

$$\frac{C(s)}{R(s)} = \frac{16}{s(s+0.8) + 16(Ks+1)}$$

$$\frac{C(s)}{R(s)} = \frac{16}{s^2 + 0.8s + 16ks + 16}$$

$$\frac{C(s)}{R(s)} = \frac{16}{s^2 + s(0.8 + 16k) + 16}$$

Compare with 2nd order System :-

$$\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{16}{s^2 + s(0.8 + 16k) + 16}$$

$$\omega_n^2 = 16$$

$$\omega_n = 4$$

$$2\xi\omega_n = 0.8 + 16k$$

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$2 \times 0.5 \times A = 0.8 + 16k.$

$A = 0.8 + 16k.$

$A - 0.8 = 16k.$

$\frac{3.2}{16} = k.$

$\boxed{K = 0.2}$

$$\frac{C(S)}{R(S)} = \frac{16}{S^2 + S(0.8 + 16K) + 16}$$

$$\frac{C(S)}{R(S)} = \frac{16}{S^2 + S[0.8 + 16(0.2)) + 16}$$

$$\frac{C(S)}{R(S)} = \frac{16}{S^2 + S(0.8 + 3.2) + 16}$$

$$\frac{C(S)}{R(S)} = \frac{16}{S^2 + 4S + 16}$$

$$C(S) = R(S) \cdot \frac{16}{S^2 + 4S + 16.}$$

For unit step input $R(S) = 1/S.$

$$C(S) = \frac{1}{S} \cdot \frac{16}{S^2 + 4S + 16}$$

$$C(S) = \frac{16}{S(S^2 + 4S + 16)}$$

$$C(S) = \frac{A}{S} + \frac{BS + C}{S(S^2 + 4S + 16)}$$

To find A, B and C.

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$$\frac{16}{s(s^2+4s+16)} = \frac{A}{s} + \frac{Bs+C}{s^2+4s+16}$$

$A = 1 \; ; \; B = -1 \; ; \; C = -4$

$$C(s) = \frac{A}{s} + \frac{Bs+C}{s^2+4s+16}$$

$$C(s) = \frac{1}{s} - \frac{(s+4)}{s^2+4s+16}$$

$$= \frac{1}{s} - \frac{(s+2+2)}{(s^2+4s+4)+12}$$

$$= \frac{1}{s} - \frac{(s+2)}{(s+2)^2+12} - \frac{2}{(s+2)^2+12}$$

$$C(t) = \frac{1}{s} - \frac{(s+2)}{(s+2)^2+(\sqrt{12})^2} - \frac{2}{\sqrt{12}} \times \frac{\sqrt{12}}{(s+2)^2+12}$$

$$L^{-1}\left\{\frac{s+a}{(s+a)^2+\omega^2}\right\} = e^{-at}\cos\omega t$$

$$L^{-1}\left\{\frac{\omega}{(s+a)^2+\omega^2}\right\} = e^{-at}\sin\omega t$$

$C(t) = L^{-1}\{C/s\}$

$$C(t) = L^{-1}\left\{\frac{1}{s} - \frac{(s+2)}{(s+2)^2+(\sqrt{12})^2} - \frac{2}{\sqrt{12}} \times \frac{\sqrt{12}}{(s+2)^2+12}\right\}$$

$$= 1 - e^{-at}\cos\sqrt{12}\,t - \frac{2}{\sqrt{12}}\, e^{-2t}\sin\sqrt{12}\,t.$$

$$C(t) = 1 - e^{-2t}\left(\cos\sqrt{12}\,t - \frac{2}{\sqrt{12}}\sin\sqrt{12}\,t\right)$$

1) $t_p = \frac{\pi}{\omega_d}$ $\quad [\because \omega_d = \omega_n\sqrt{1-\xi^2}]$ $\quad \xi = 0.5 \quad \omega_n = 4.$

$$= \frac{\pi}{4\sqrt{1-(0.5)^2}}$$

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$t_p = 0.907$ Sec

rad - mode.

2) $t_r = \frac{\pi - \theta}{\omega_d} = \frac{\pi - 1.047}{\omega_n\sqrt{1-\xi^2}}$

$\theta = \tan^{-1}\left\{\frac{\sqrt{1-\xi^2}}{\xi}\right\}$

$t_r = 0.6469$ Sec

$\theta = 1.047$.

3) Settling time (t_s)

$t_s = 3T = 3 \times \frac{1}{\xi\omega_n} = \frac{3}{\xi\omega_n} = \frac{3}{0.5 \times 4} = 1.5$ Sec

(for 5% error)

$t_s = 4T = 4 \times \frac{1}{\xi\omega_n} = \frac{4}{0.5 \times 4} = 2$ Sec.

(for 2% error)

4) Maximum peak overshoot.

$\%\ M_p = \left\{e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}\right\} \times 100.$

$\%\ M_p = 0.163 \times 100.$

$\%\ M_p = 16.3.$

2) A unity feedback system characterised by the following open loop transfer function.

$G(s) = \frac{0.4s + 1}{s(s + 0.6)}$ Determine transient Response for unit step input Evaluate Peak time & Maximum Overshoot

Soln:-

$H(s) = 1$; $G(s) = \frac{0.4s + 1}{s(s + 0.6)}$

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To find $c(t) = ?$ $t_p = ?$ $\%M_p = ?$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$G(s) = \frac{0.4s + 1}{s(s + 0.6)}$$

$$H(s) = 1.$$

$$\frac{C(s)}{R(s)} = \frac{\frac{0.4s+1}{s(s+0.6)}}{1 + \frac{0.4s+1}{s(s+0.6)}(1)} = \frac{\frac{0.4s+1}{s(s+0.6)}}{\frac{s(s+0.6) + 0.4s + 1}{s(s+0.6)}}$$

$$\frac{C(s)}{R(s)} = \frac{0.4s+1}{s(s+0.6)+0.4s+1} = \frac{0.4s+1}{s^2+0.6s+0.4s+1}$$

$$\frac{C(s)}{R(s)} = \frac{0.4s+1}{s^2+s(0.6+0.4)+1}$$

Compare with 2nd Order

$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{0.4s+1}{s^2+s(0.6+0.4)+1}$$

$$\omega_n^2 = 0.4s + 1.$$

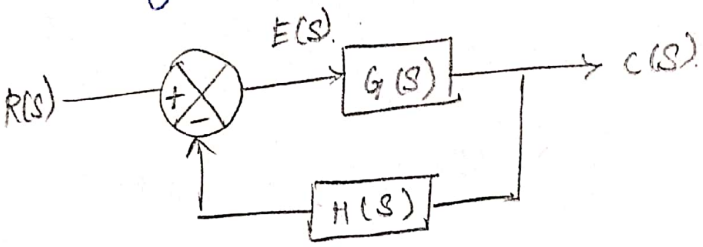
$$\omega_n = 1.748.$$

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Steady state error :

The steady state error is the value of error signal which t tends to Infinity.

Steady state is Measure the System accuracy



R(S) = Input signal C(S) - output signal

C(S) H(S) - Feedback signal E(S) - Error signal.

Error signal $E(S) = R(S) - C(S)\,H(S)$ — ①.

output signal $C(S) = E(S)\,G(S)$ — ②.

Sub ② in ①.

$$E(S) = R(S) - E(S)\,G(S)\,H(S)$$

$$E(S) + E(S)\,G(S)\,H(S) = R(S)$$

$$E(S)\left[1 + G(S)\,H(S)\right] = R(S)$$

$$E(S) = \frac{R(S)}{1+G(S)H(S)}$$

$$E(S) = \frac{1}{1+G(S)H(S)} \cdot R(S) \qquad \left[\because F(S) = \frac{1}{1+G(S)H(S)}\right]$$

$$E(S) = F(S)\,R(S)$$

To find the Steady State error in two Methods:

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case (i) error signal in time Domain

case (ii) Generalized error co-effiient.

case (i) error signal in time Domain

$$e_{ss} = \lim_{t \to \infty} e(t).$$

$$e(t) = \lim_{s \to 0} sE(s)$$

$$E(s) = \frac{R(s)}{1 + G(s)H(s)}$$

case (ii) Generalized error co-effiient.

$$e_{ss} = \lim_{t \to \infty} e(t)$$

$$e(t) = c_0 r(t) + \frac{c_1}{1!} r^{o}(t) + \frac{c_2}{2!} r^{oo}(t) + \frac{c_3}{3!} r^{ooo}(t) + \cdots \frac{c_n}{n!} r^{n}(t).$$

$$c_0 = \lim_{s \to 0} F(s)$$

$$c_1 = \lim_{s \to 0} \frac{d}{ds} F(s)$$

$$c_2 = \lim_{s \to 0} \frac{d^2}{ds} F(s)$$

$$\left[\because F(s) = \frac{1}{1 + G(s)H(s)} \right.$$

$$\left. r(t) = L^{-1}\{r(s)\} \right]$$

$$r^{o}(t) = \frac{d}{dt} r(t)$$

$$r^{oo}(t) = \frac{d}{dt} r^{o}(t).$$

Static Error Constant :-

(Type 0) : Positional error constant $K_p = \lim_{s \to 0} G(s)H(s)$ (Input is step signal)

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(Type 1) : Velocity error constant

$$K_r = \lim_{s \to 0} s\,G(s)\,H(s)$$ (Input is Ramp signal)

(Type 2): Acceleration error constant

$$K_a = \lim_{s \to 0} s^2\,G(s)\,H(s)$$ (Input is parabolic signal)

Static error for various type of systems :-

Error constant	Type of System.			
	0	1	2	3
K_p	constant	∞	∞	∞
K_r	0	constant	∞	∞
K_a	0	0	constant	∞

Steady State Error for Various type of

Input signal	Type of System			
	0	1	2	3
Unit Step	$\frac{1}{1+K_p}$	0	0	0
Unit Step	∞	$\frac{1}{K_v}$	0	0
Unit parabola	∞	∞	$\frac{1}{K_a}$	0.

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Type and order of System.

1) $G(s) = \frac{s}{s(s+1)(s+2)}$

Type of System: 1 (highest power of s)

Order of System: 3 (Multiples of s)

2) $G(s) = \frac{s}{s^2(s+1)}$

Type of System :- 2 (highest power of s)

Order of System :- 3 (Multiples of s)

1. For a Unity feedback System the open loop Transfer Function is $G(s) = \frac{10(s+2)}{s^2(s+1)}$ Find

a) Position, velocity & acceleration error constant

b) Steady state error when the Input R(s) is

$$R(s) = \frac{3}{s} - \frac{2}{s^2} + \frac{1}{3s^3}$$

Sol :-

$$G(s) = \frac{10(s+2)}{s^2(s+1)}$$

Type :- 2 $\quad H(s) = 1.$

Order :- 3

$$K_p = \lim_{s \to 0} G(s)H(s)$$

$$K_r = \lim_{s \to 0} G(s)H(s)$$

$K_p = \infty$

$K_r = \infty$

$K_a =$ constant.

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$$K_a = \lim_{s \to 0} s^2 G(s) H(s)$$

$$= \lim_{s \to 0} \frac{10(s+2)}{s^2(s+1)} \quad (1)$$

$$= \frac{10(0+2)}{0(0+1)}$$

$$= \frac{20}{0}$$

$$K_p = \infty$$

$$K_r = \lim_{s \to 0} s\, G(s) H(s)$$

$$= \lim_{s \to 0} s \frac{10(s+2)}{s^2(s+1)} \quad (1)$$

$$= \frac{10(0+2)}{0(0+1)}$$

$$= \frac{20}{0}$$

$$K_r = \infty$$

$$K_a = \lim_{s \to 0} s^2 G(s) H(s)$$

$$= \lim_{s \to 0} s^2 \frac{10(s+2)}{s^2(s+1)}$$

$$= \frac{10(0+2)}{(0+1)} = \frac{10(2)}{1}$$

$$K_a = 20.$$

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To find steady state error.

Method I :- Generalized error constant.

$$e_{ss} = \lim_{t \to \infty} e(t)$$

$$e(t) = r(t)\, c_0 + r^{o}(t)\frac{c_1}{1!} + r^{oo}(t)\frac{c_2}{2!} + r^{ooo}(t)\frac{c_3}{3!} + \ldots r^{n}(t)\frac{c_n}{n!}$$

In given :-

$$R(s) = \frac{3}{s} - \frac{2}{s^2} + \frac{1}{3s^3}$$

$$r(t) = L^{-1}\{R(s)\}$$

$$= L^{-1}\left\{\frac{3}{s} - \frac{2}{s^2} + \frac{1}{3s^3}\right\}$$

$$= 3 - 2t + \frac{1}{3}\frac{t^2}{2}$$

$$r(t) = 3 - 2t + \frac{t^2}{6}$$

$$r(t) = 3 - 2t + \frac{t^2}{6}$$

$$r^{o}(t) = \frac{d}{dt} r(t)$$

$$= \frac{d}{dt}\left[3 - 2t + \frac{t^2}{6}\right] \Rightarrow r^{o}(t) = \left[-2 + \frac{2t}{6}\right].$$

$$r^{o}(t) = -2 + \frac{t}{3}.$$

$$r^{oo}(t) = \frac{d}{dt} + r^{o}(t)$$

$$= \frac{d}{dt}\left(-2 + t(3)\right)$$

$$r^{oo}(t) = 1/3.$$

$$c_0 = \lim_{s \to 0} F(s)$$

$$c_1 = \lim_{s \to 0} \frac{d}{ds} F(s)$$

$$c_2 = \lim_{s \to 0} \frac{d^2}{ds^2} F(s)$$

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$$r^{ooo}(t) = \frac{d}{dt} r^{oo}(t)$$

$$= \frac{d}{dt}\left(\frac{1}{3}\right)$$

$$r^{ooo}(t) = 0$$

$$C_0 = \lim_{s \to 0} F(s)$$

$$C_0 = \lim_{s \to 0} \frac{1}{1 + G(s)H(s)} \Rightarrow \lim_{s \to 0} \frac{1}{1 + \frac{10(s+2)}{s^2(s+1)}} \cdot 1$$

$$= \lim_{s \to 0} \frac{1}{\frac{s^2(s+1) + 10(s+2)}{s2(s+1)}}$$

$$C_0 = \lim_{s \to 0} \frac{s^2(s+1)}{s^2(s+1) + 10(s+2)}$$

$$C_0 = \frac{0(0+1)}{0(0+1) + 10(0+2)}$$

$$C_0 = \frac{0}{20}$$

$$C_0 = 0$$

$$C_1 = \lim_{s \to 0} sF(s)$$

$$= \frac{d}{ds}\left[\frac{s^3 + s^2}{s^3 + s^2 + 10s + 20}\right]$$

$$\frac{u}{v} = \frac{vu' - uv'}{v^2}$$

$$= \lim_{s \to 0} \left[\frac{(s^3 + s^2 + 10s + 20)(3s^2 + 2s) - (s^3 + s^2)(3s^2 + 2s + 10)}{(s^3 + s^2 + 10s + 20)^2}\right.$$

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$$= \lim_{s \to 0} \left[\frac{3s^5 + 2s^4 + 3s^4 + 2s^3 + 30s^3 + 60s^2 + 40s - 3s^5 - 2s^4 - 10s^3 - 3s + 20s^2 - 2s^3 - 10s^2}{(s^3 + s^2 + 10s + 20)^2} \right]$$

$$C_1 = \lim_{s \to 0} \left[\frac{20s^3 + 70s^2 + 40s}{(s^3 + s^2 + 10s + 20)^2} \right]$$

$$C_1 = \frac{0}{(20)^2}$$

$$\boxed{C_1 = 0.}$$

$$C_2 = \frac{d^2}{ds^2} F(s)$$

$$= \frac{d}{ds} \left[\frac{d}{ds} F(s) \right]$$

$$= \frac{d}{ds} \left[\frac{20s^3 + 70s^2 + 40s}{(s^3 + s^2 + 10s + 20)^2} \right]$$

$$\frac{u}{v} = \frac{vu' - uv'}{v^2}$$

$$= \lim_{s \to 0} \left[\frac{(s^3 + s^2 + 10s + 20)^2 (60s^2 + 140s + 40) - (20s^3 + 70s^2 + 40s) 2 \times (s^3 + s^2 + 10s + 20)(3s^2 + 2s + 10)}{(s^3 + s^2 + 10s + 20)^4} \right]$$

$$= \frac{20^2 \times 40}{(20)^4} = \frac{20 \times 20 \times 40}{20 \times 20 \times 20 \times 20} = \frac{40}{400} = \frac{4}{40} = \frac{1}{10.}$$

$$C_2 = \frac{1}{10.}$$

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Error Signal:

$$e(t) = c_0 r(t) + \frac{c_1}{1!} r^{0}(t) + \frac{c_2}{2!} r^{00}(t) + \frac{c_3}{3!} r^{000}(t)$$

$$e(t) = 0 \times \left(\frac{1}{3} - 2t + \frac{t^2}{6}\right) + 0\left(-2 + \frac{t}{3}\right) + \left(\frac{1}{10} + \frac{1}{2} \times \frac{1}{\infty}\right)$$

$$\boxed{e(t) = \frac{1}{60}}$$

$$e_{ss} = \lim_{t \to \infty} e(t)$$

$$= \lim_{t \to \infty} \frac{1}{60}$$

$$e_{ss} = \frac{1}{60}.$$

Method II :

Error Signal in S-domain.

$$E(S) = \frac{R(S)}{1 + G(S)H(S)}$$

$$\left[\because G(S) = \frac{10(S+2)}{S^2(S+1)}\right.$$

$$\left. R(S) = \frac{3}{S} - \frac{2}{S^2} + \frac{1}{3S^3}\right]$$

$$= \frac{\frac{3}{2} - \frac{2}{S^2} + \frac{1}{3S^3}}{1 + \frac{10(S+2)}{S^2(S+1)}}$$

$$= \frac{\frac{3}{S} - \frac{2}{S2} + \frac{1}{3S^3}}{\frac{S^2(S+1) + 10(S+2)}{S^2(S+1)}}$$

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$$= \frac{s^2(s+1)\left[\frac{3}{s} - \frac{2}{s^2} + \frac{1}{3s^3}\right]}{s^2(s+1)+10(s+2)} = \frac{s^2(s+1)\left[\frac{9s^2-6s+1}{3s^3}\right]}{s^2(s+1)+10(s+2)}$$

$$= \frac{(s+1)(9s^2-6s+1)}{3s\left[s^2(s+1)+10(s+2)\right]}$$

$$e_{ss} = \lim_{t\to\infty} e(t)$$

$$e(t) = \lim_{s\to 0} sE(s)$$

$$= \lim_{s\to 0} s\,\frac{(s+1)(9s^2-6s+1)}{3s\left[s^2(s+1)+10(s+2)\right]}$$

$$= \frac{(0+1)(0-0+1)}{3\left[0^2(0+1)+10(0+2)\right]} = \frac{1(1)}{3[20]} = \frac{1}{60}$$

$$e(t) = \frac{1}{60}$$

$$e_{ss} = \lim_{t\to\infty} \frac{1}{60}$$

$$e_{ss} = \frac{1}{60}$$

Evaluate the open loop transfer Function with unity feedback System. $G(s) = \frac{10}{s(0.1s+1)}$ calculate static error constant and steady state error with Input

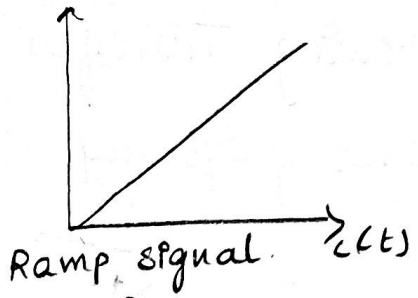
$$r(t) = a_0 + a_1 t + \frac{a_2}{2!} t^2.$$

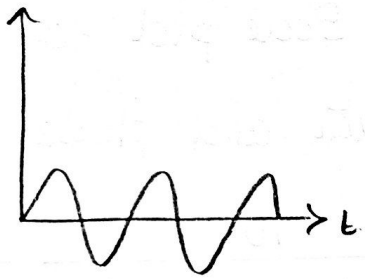
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UNIT- III

Frequency Response Analysis.

The Response of a System for the Sinusoidal Input is called Sinusoidal response.





$$T(S) \xrightarrow{S=j\omega} T(j\omega).$$

The frequency domain Transfer Function $T(j\omega)$ is a complex function. Hence it can be Separated into Magnitude and phase function.

$$G(S)H(S) \xrightarrow{S=j\omega} G(j\omega)\,H(j\omega)$$

$$= |G(j\omega)\,H(j\omega)| \leftarrow \text{Magnitude functions.}$$

$$= \angle G(j\omega)H(j\omega) \leftarrow \text{phase Functions.}$$

Procedure for Bode plot :-

Step 1 :- Replace the format $(1+ST)$.

Step 2 :- Replace S by $j\omega$.

Step 3:- To find corner frequency.

Step 4:- Tabulation for Magnitude plot.

Step 5:- Find out the Magnitude value.

Step 6:- Tabulation for phase plot.

1) Plot the Bode plot for the following Transfer function obtain gain and phase cross over frequency.

$$G(s) = \frac{10}{s(1+0.4s)(1+0.1s)}$$

$$\frac{1}{s+2} = \frac{1}{2(\frac{s}{2}+1)} = \frac{1}{2(1+\cdots)}$$

$$\therefore \frac{1}{(4+4s)} = \frac{1}{4(1+\frac{4}{4}s)}$$

$$= \frac{1}{4(1+s)}$$

Step 1:- Replace $(1+sT)$.

Step 2:- Replace s by $(j\omega)$

$$G(j\omega) = \frac{10}{j\omega(1+0.4j\omega)(1+0.1j\omega)}$$

Step 3:- To find corner frequency.

$$\omega_c = \frac{1}{T} \quad \omega_c \rightarrow \text{corner frequency.}$$

$$G(j\omega) = \frac{10}{j\omega(1+0.4j\omega)(1+0.1j\omega)}$$

$$\omega_{c_1} = \frac{1}{0.4} = 2.5 \text{ rad/sec}$$

$$\omega_{c_2} = \frac{1}{0.1} = 10. \text{ rad/sec}$$

Formula

Slope

$\frac{1}{j\omega} = -20$

$\frac{1}{(j\omega)^2} = -40$

$j\omega = +20$

$(j\omega)^2 = +40.$

Step 4 :- Tabulation for Magnitude plot :-

Term	Corner frequency	Slop in db/sec	change in slope db/sec.
$\frac{10}{j\omega}$	–	-20	–
$\frac{1}{1+0.4j\omega}$	$\omega_{c_1} = 2.5$	-20	$-20-20 = -40$.
$\frac{1}{1+0.1j\omega}$	$\omega_{c_2} = 10$	-20	$-40-20 = -60$.

$(j\omega) = 20$
$(j\omega)^2 = 40$
$\frac{1}{j\omega} = -20$
$\frac{1}{(j\omega)^2} = -40$

Step 5 :- To find the Magnitude value :-

$\omega_l < \omega_{c_1}$, $\omega_h > \omega_{c_2}$ ω_l — Lower frequency
ω_h — Higher frequency

$\omega_l = 0.5$

$\omega_{c_1} = 2.5$

$\omega_{c_2} = 10$

$\omega_h = 50.$ $A = |G(j\omega)|$

$\omega = \omega_l = 0.5$; $A = 20 \log$ term $= 20 \log\left(\frac{10}{j\omega}\right) = 20 \log\left(\frac{10}{0.5}\right)$

$A = 26$ db.

$\omega = \omega_{c_1} = 2.5$; $A = 20 \log\left(\frac{10}{j\omega}\right) = 20 \log\left(\frac{10}{2.5}\right)$

$A = 12$ db.

$\omega = \omega_{c_2} = 10$; $A = [$slope from ω_{c_1} to $\omega_{c_2} \times \log\left(\frac{\omega_{c_2}}{\omega_{c_1}}\right)] +$ A at $(\omega = \omega_{c_1})$

$$A = \left[-40 \times \log\left(\frac{10}{2.5}\right)\right] + 12.$$

$$\boxed{A = -12 \text{ db}}$$

$\omega = \omega_n = 50$;

$$A = [\text{Slop from } \omega_{c_2} \text{ to } \omega_n \times \log\left(\frac{\omega_n}{\omega_{c_2}}\right)] + A \text{ (at } \omega = \omega_{c_2})$$

$$= [-60 \times \log\left(\frac{50}{10}\right)] + (-12).$$

$$A = -54 \text{ db}.$$

$$G(s) = \frac{10}{s(1+0.4s)(1+0.1s)}$$

Magnitude

$\omega = \omega_l = \underline{0.5}$ $A = \underline{20 \text{ db}}$

$\omega = \omega_{c_1} = \underline{2.5}$; $A = \underline{12 \text{ db}}$.

$\omega = \omega_{c_2} = \underline{16}$, $A = \underline{-12 \text{ db}}$.

$\omega = \omega_n = 50$, $A = -54$.

Formula

Magnitude

$\frac{1}{j\omega} = 20$

$\frac{1}{(j\omega)^2} = +40$.

$j\omega = +40$

$(j\omega)^2 = +40$

Step b :- To find phase value

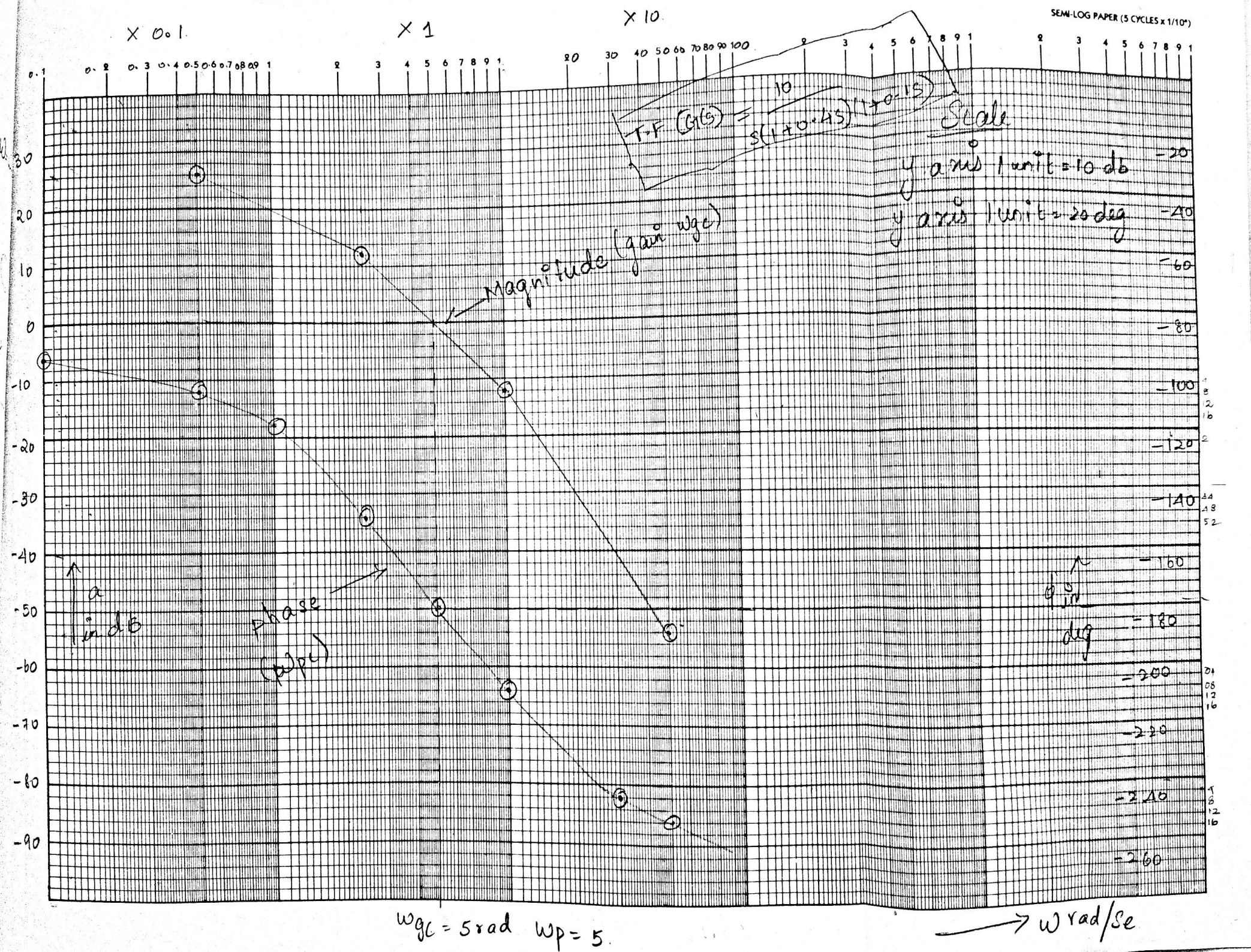
$$G(j\omega) = \frac{10}{j\omega(1+0.4j\omega)(1+0.1j\omega)}$$

$$\phi = -90 - \tan^{-1} 0.4\omega - \tan^{-1} 0.1\omega$$

$\left[\frac{1}{1+j\omega} = -\tan^{-1}\omega,\ 1+j\omega = \tan^{-1}\omega,\ \frac{1}{j\omega} = -90\right.$

$\left.\frac{1}{(j\omega)^2} = -180,\ j\omega = 90,\ (j\omega)^2 = 180\right]$

ω rad/sec	$\tan^{-1}(0.4\omega)$	$\tan^{-1}0.1\omega$	$\phi = -90 - \tan^{-1}0.4\omega - \tan^{-1}0.1\omega$
0.1	2.291	0.571	$\phi = -93$
0.5	11.310	2.8553	$\phi = -104$
1	21.801	5.7106	$\phi = -118$
2.5	45.000	14.27	$\phi = -150$
5	63.435	28 55	$\phi = -180$
10	75.964	57.105	$\phi = -210$
30	85.236	171.31	$\phi = -246$
50	87.138	285.52	$\phi = -255$


SEMI-LOG PAPER (5 CYCLES x 1/10⁴)
× 0.1
× 1
× 10
T.F G(s) = 10 / s(1+0.4s)(1+0.1s)
Scale
y axis 1 unit = 10 db
y axis 1 unit = 20 deg
Magnitude (gain wgc)
Phase (pm)
a in dB
ϕ in deg
wgc = 5 rad wp = 5
w rad/se

2) For the transfer function $G(s) = \dfrac{5(1+2s)}{(1+4s)(1+0.25s)}$

Draw the Bode plot.

Step 1:- Format in $(1+sT)$

Step 2:- Replace $s = j\omega$.

$$G(s) = \frac{5(1+2s)}{(1+4s)(1+0.25s)}$$

$$G(j\omega) = \frac{5(1+2j\omega)}{(1+4j\omega)(1+0.25j\omega)}$$

Formula

$\frac{1}{j\omega} = -20$
$\frac{1}{(j\omega)^2} = -40$
$(j\omega) = 20$
$(j\omega)^2 = +40$

Step 3 :- To find corner frequency.

$$\omega_c = \frac{1}{T}$$

$$\omega_{c_2} = \frac{1}{2} = 0.5 \qquad \omega_{c_1} = \frac{1}{4} = 0.25.$$

$$\omega_{c_2} = \frac{1}{0.25} = 4.$$

Step 4 :- Tabulation for Magnitude plot.

Term	corner frequency	Slop in db /sec	change in slope.
5	-	-	-
$\frac{1}{1+j4\omega}$	$\omega_{c_1} = 0.25$	-20	$0-20 = -20.$
$(1+j2\omega)$	$\omega_{c_2} = 0.5$	20	$-20+20 = 0$
$\frac{1}{(1+j0.25\omega)}$	$\omega_{c_3} = 4$	-20	$0-20 = -20.$

Step 5 :- To find Magnitude value

$\omega_l < \omega_{c_1}$, $\omega_h > \omega_{c_3}$; $\omega_l = 0.1$ $\omega_{c_3} = 4$

$\omega_{c_1} = 0.25$ $\omega_h = 50.$

$\omega_{c_2} = 0.5$

$\omega = \omega_l = 0.1$ $A = 20 \log (\text{term}) = 20 \log (5)$.

$\omega = \omega_l$; $A = 14$ db.

$\omega = \omega_{c_1} = 0.25$; $A = 20 \log 5$.

$A = 14$ db.

$\omega = \omega_{c_2} = 0.5$; $A = [\text{Slope from } \omega_{c_1} \text{ to } \omega_{c_2} \times \log \left(\frac{\omega_{c_2}}{\omega_{c_1}}\right)]$

$+ A \text{ (at } \omega = \omega_{c_1})$

$= \left[20 \times \log \left(\frac{0.5}{0.25}\right)\right] + 14$

$= [-20 \times 0.979$

$= 8$ db.

$\omega = \omega_{c_3} = 4$; $A = [\text{Slope from } \omega_{c_2} \text{ to } \omega_{c_3} \times \log \left(\frac{\omega_{c_3}}{\omega_{c_2}}\right)] + A \text{ (at } \omega = \omega_{c_2})$

$A = \left[0 \times \log \left(\frac{4}{0.5}\right)\right] + 8$.

$A = 8$ db.

$\omega = \omega_{c_h} = 50$; $A = [\text{Slope from } \omega_{c_3} \text{ to } \omega_h \times \log \left(\frac{\omega_{ch}}{\omega_{c_3}}\right)] + A \text{ (at } \omega = \omega_{c_3})$

$= \left[-20 \times \log \left(\frac{50}{4}\right)\right] + 8$.

$A = -14$ db.

Magnitude value

$\omega_l = 0.1 = 14$ db.

$\omega c_1 = 0.25 = 14$ db.

$\omega c_2 = 0.5 = 8$ db.

$\omega c_3 = 4 = 8$ db.

$\omega_n = 50 = -14$ db.

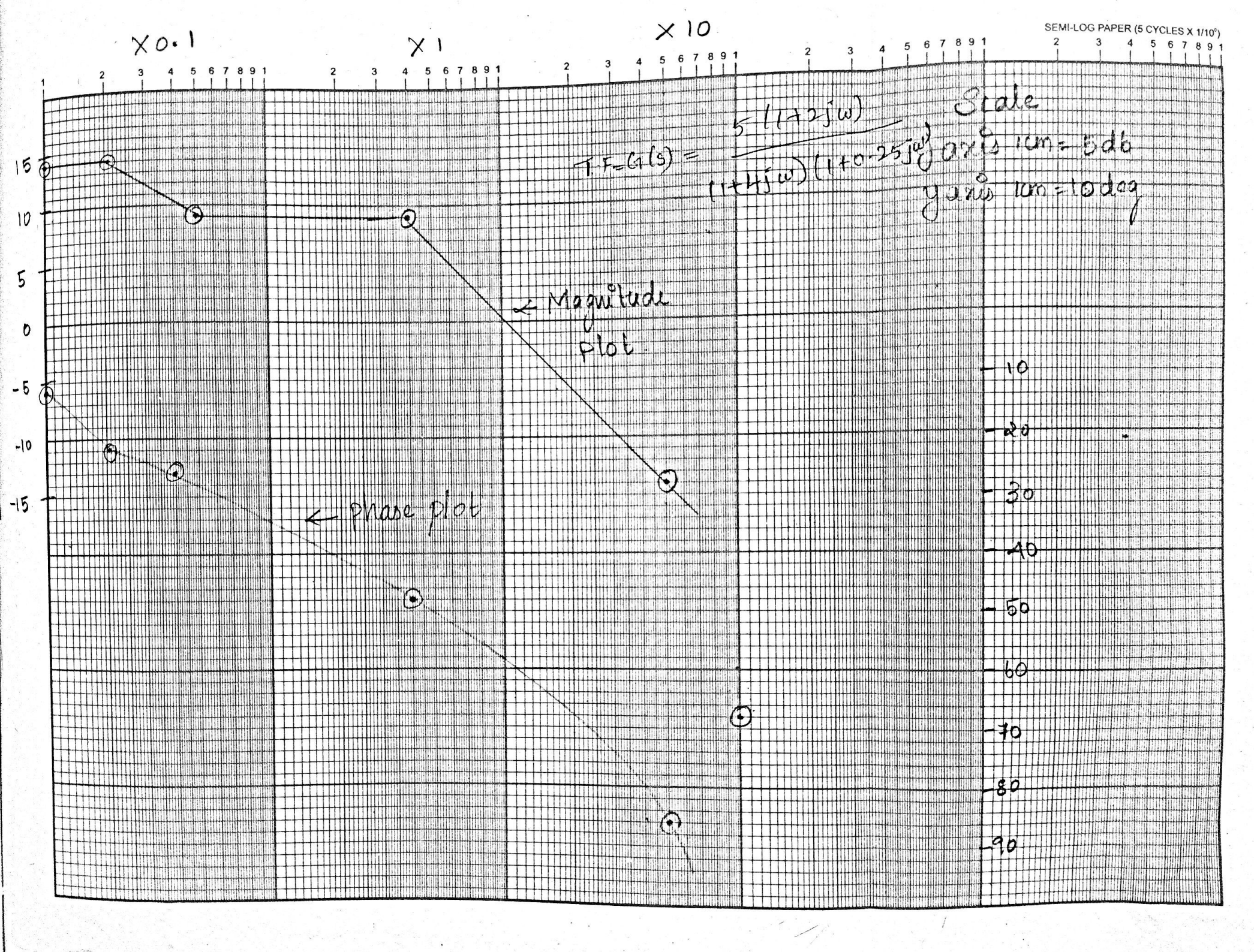
Step 6 :- To find the phase value :-

$$G(j\omega) = \frac{5(1+2j\omega)}{(1+4j\omega)(1+0.25j\omega)}$$

$$\left[(1+j\omega) = \tan^{-1}\omega \; ; \; \frac{1}{1+j\omega} = -\tan^{-1}\omega \; ; \; \frac{1}{1+j2\omega} = \tan^{-1}0.25\omega\right]$$

$$\boxed{\phi = \tan^{-1}2\omega - \tan^{-1}4\omega - \tan^{-1}0.25\omega}$$

ω	$\tan^{-1}2\omega$	$\tan^{-1}4\omega$	$\phi = \tan^{-1}2\omega - \tan^{-1}4\omega - \tan^{-1}0.25\omega$
0.1	1.1974	0.3805	-12
0.25	0.4636	0.7854	-22
0.5	0.7854	1.1071	-26
4	1.4464	1.5084	-49
10	1.5208	1.5458	-70
50	1.5608.	1.5658.	-86.


SEMI-LOG PAPER (5 CYCLES X 1/10°)
X0.1
X1
X10
1. F=G(s) = 5(1+2jω) / (1+4jω)(1+0.25jω)
Scale
y axis 1cm = 5db
y axis 1cm = 10deg
Magnitude plot
phase plot
15
10
5
0
-5
-10
-15
-10
-20
-30
-40
-50
-60
-70
-80
-90

4) Sketch the Bode plot for the following Transfer function and determine phase Margin & gain Margin.

$$G(s) = \frac{75(1+0.2s)}{s(s^2+16s+100)}$$

Soln :-

On comparing the quadratic Factor we determine the value of ξ and ω_n.

$$s^2+16s+100 = s^2+2\xi\omega_n s+\omega_n^2$$

$\omega_n^2 = 100$; $2\zeta\omega_n = 16$

$\omega_n = 10$ $\quad 2\zeta \times 10 = 16$

$20\zeta = 16.$

$\zeta = 0.8.$

$$G(s) = \frac{75(1+0.2s)}{s(s^2+16s+100)}$$

$$= \frac{75(1+0.2s)}{s \times 100\left(\frac{s^2}{100}+\frac{16s}{100}+1\right)}$$

$$G(s) = \frac{75(1+0.2s)}{100s(0.01s^2+0.16s+1)}$$

$$G(s) = \frac{0.75(1+0.2s)}{s(0.01s^2+0.16s+1).}$$

Step 1 :-

Replace $s = j\omega$.

$$G(j\omega) = \frac{0.75(1+0.2j\omega)}{j\omega[0.01(j\omega)^2+0.16j\omega+1].} \qquad [\because j^2 = -1]$$

$$G(j\omega) = \frac{0.75(1+j0.2\omega)}{j\omega[1-0.01\omega^2+j0.16\omega]}$$

Step 2 :- To find corner frequency

$\omega_c = \frac{1}{T}$

$\omega_c = \frac{1}{0.2}$

$\omega_{c_1} = 5$ rad/sec

$\omega_{c_2} = \omega_n = 10$

$\omega_{c_2} = 10$ rad/sec.

Step 3:- Magnitude plot Tabulation :-

Term	corner frequency	slope in db	change in slope.
$\frac{0.75}{j\omega}$	–	-20	–
$(1+j0.2\omega)$	$\omega_{c_1} = 5$ rad/sec	$+20$	$-20+20=0$.
$\frac{1}{(1-0.01\omega^2+j0.1\omega)}$	$\omega_{c_2} = 10$	-40	$0-40=-40$.

$\frac{1}{j\omega} = -20$

$\frac{1}{(j\omega)^2} = -40$

$j\omega = 20$

$(j\omega)^2 = 40$.

Step 4:- To find Magnitute value :-

$\omega_l = 0.5$

$\omega_{c_1} = 5$

$\omega_{c_2} = 10$

$\omega_h = 50$

$\omega_l < \omega_{c_1}$

$\omega_h > \omega_{c_2}$.

$\omega = \omega_l = 0.5$, $A = 20 \log$ term

$= 20 \log \left(\frac{0.75}{j\omega}\right)$

$= 20 \log\left(\frac{0.75}{0.5}\right)$

$= 3db.$

$\omega - \omega c_1 = 5, \quad A = 20 \log\left(\frac{0.75}{j\omega}\right)$

$= 20 \log\left(\frac{0.75}{5}\right)$

$= -16 \text{ db}.$

$\omega = \omega c_2 = 10;$

$A = [\text{Slope from } \omega c_1 + \omega c_2 \times \log\left(\frac{\omega c_2}{\omega c_1}\right)] + A \text{ (at } \omega - \omega c_1)$

$= [0 * \log\left(\frac{10}{5}\right) + (-16)$

$A = -16$

$\omega = \omega_n = 50$

$A = [\text{Slope from } \omega c_3 + \omega_n * \log\left(\frac{\omega c_1}{\omega c_2}\right) + A_{(\text{at } \omega = \omega_2)}$

$= 40\left[-40 \times \log\left(\frac{50}{10}\right)\right] + (-16)$

$A = -44 \text{ db}.$

$\left[\frac{1}{j\omega} = -90\right.$

$\frac{1}{(j\omega)^2} = -180$

$\frac{1}{1+j\omega} = \tan^{-1}\omega$

$\left.1 + j\omega = \tan^{-1}\omega\right]$

Step 5:- To find phase value.

$\phi = -90 + \tan^{-1} 0.2\omega - \tan^{-1}\left(\frac{0.16\omega}{1 - 0.01\omega^2}\right)$

ω	$\tan^{-1}0.2\omega$	$\tan^{-1}\left(\frac{0.16\omega}{1-0.01\omega^2}\right)$	$\phi = -90 + \tan^{-1}0.2\omega - \tan^{-1}\left(\frac{0.16\omega}{1-0.01\omega^2}\right)$
0.5	5.7	4.6	$-88.9 = -89$
1	11.3	9.2	$-87.9 = -88$
5	45	47	$-91.8 = -92$
10	63	$+90$	$-116.6 = -116.$
20	76	$-46.8 + 180 = 133.2$	$-141.3 \approx -148.$
50	84	$-18.4 + 180 = 161.6$	-168
100	87	$-9.2 + 180 = 170.8$	-174

Magnitude plot.

$\omega = \omega_L = 0.5 = 3db.$

$\omega = \omega_{c1} = 5 = -16db$

$\omega = \omega_{c2} = 10 = -16db$

$\omega = \omega_h = 50 = -44db.$

phase plot

$0.5 = -89.$ $\quad 50 = -168.$

$1 = -88$ $\quad 100 = -174.$

$5 = -92$

$10 = -117$

$20 = -148.$

Phase cross over frequency (ω_{pc})

-180° cut in phase plot.

gain cross over frequency

0° cut in Magnitude plot.

Phase Margin (γ)

$$\gamma = 180^\circ + \phi_{gc}$$

$\phi_{gc} = 0^\circ$ cut in Magnitude plot take the value in Phase plot.

Gain Margin (Kg).

$$K_a = \frac{1}{G(j\omega_{pc})} \quad ; \omega_{pc} = -180^\circ \text{ cut in phase plot}$$

take the value in Magnitude plot.

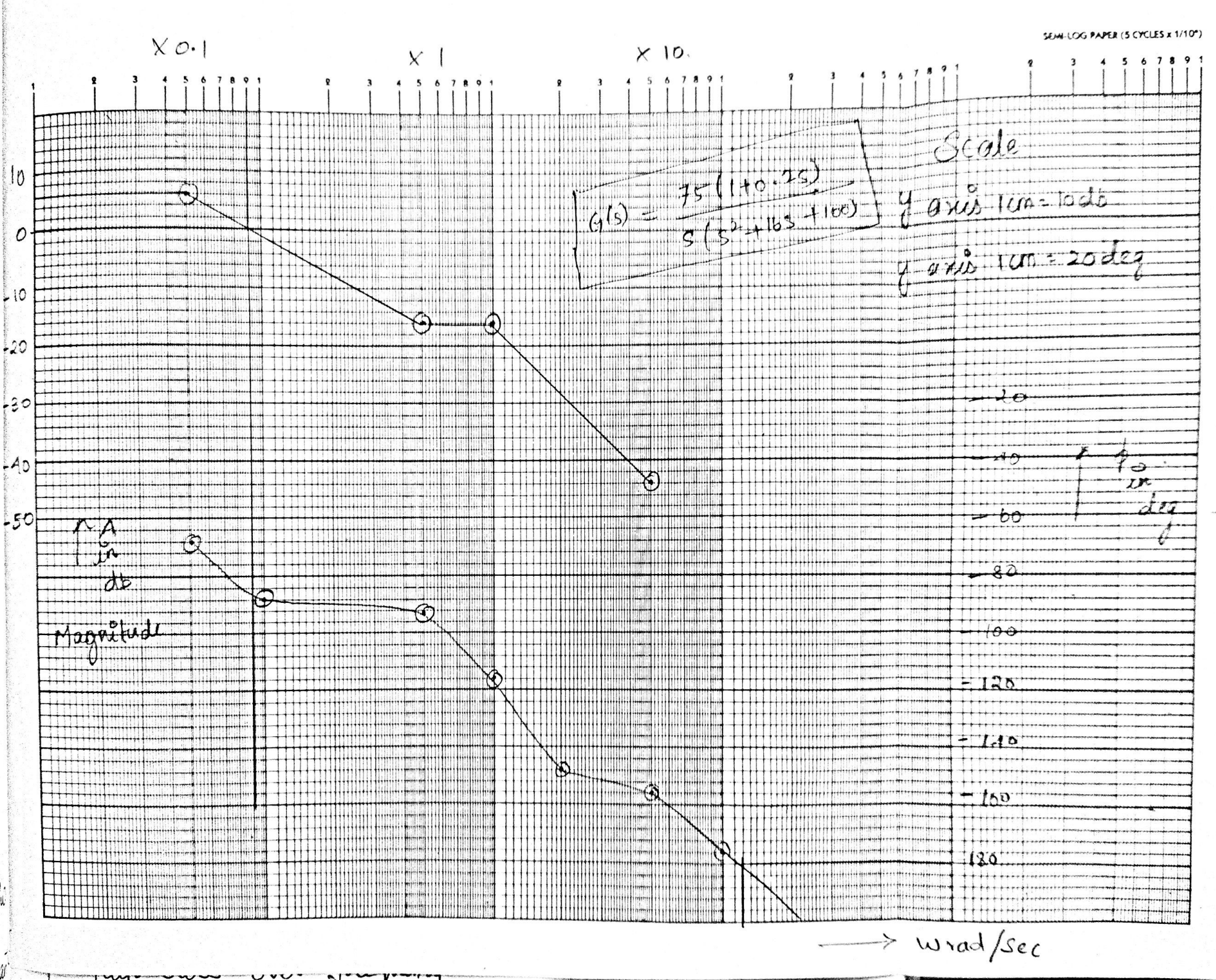
In graph $\omega_{gc} = 9.$

$\omega_{pc} = 100.$

$\gamma = 180^\circ + \phi_{gc}.$

$= 180 - 88$

$\gamma = 92.$


SEMI-LOG PAPER (5 CYCLES x 1/10")
X 0.1
X 1
X 10
$G(s) = \frac{75(1+0.2s)}{s(s^2+16s+100)}$
Scale
y axis 1cm = 10 db
y axis 1cm = 20 deg
A in db
Magnitude
10
0
-10
-20
-30
-40
-50
-20
-40
-60
-80
-100
-120
-140
-160
-180
ϕ in deg
ϕ
ω rad/sec

4) Given $G(s) = \frac{Ke^{-0.2s}}{s(s+2)(s+8)}$ Find K show that System is Stable with Bode plot.

a) Gain Margin equal to 2db.

b) phase Margin to 45°.

Soln :-

$K = 1$

$$G(s) = \frac{e^{-0.2s}}{s(s+2)(s+8)}$$

Step 1 :-

$$G(s) = \frac{e^{-0.2s}}{s(s+2)(s+8)}$$

$$G(s) = \frac{e^{-0.2s}}{s \times 2\left(\frac{s}{2}+1\right) \times 8\left(\frac{s}{8}+1\right)}$$

$$= \frac{e^{-0.2s}}{s \times 16 (1+0.5s)(1+0.125s)}$$

$$G(s) = \frac{0.063\, e^{-0.2s}}{s(1+0.5s)(1+0.125s)}$$

Step 2 :-

$s \rightarrow j\omega$

$$G(j\omega) = \frac{0.063\, e^{-0.2j\omega}}{j\omega(1+0.5j\omega)(1+0.125j\omega)}$$

Step 3 :-

To find corner frequency.

$$\omega_c = \frac{1}{T}$$

$$\omega_{c_1} = \frac{1}{0.5} = 2 \text{ rad/sec}$$

$$\omega_{c_2} = \frac{1}{0.125} = 8 \text{ rad/sec}.$$

Step 1 :- Tabulation for Magnitude plot.

Term	Corner frequency	Slop in db	change in slope.
$\frac{0.063}{j\omega}$	-	-20	-
$\frac{1}{1+0.5j\omega}$	$\omega_{c_1} = 2$ rad/sec	-20	-20-20 = -40
$\frac{1}{1+0.125j\omega}$	$\omega_{c_2} = 8$ rad/sec	-20	-40-20 = -60

$\omega_1 = 0.5$

$\omega_{c_1} = 2.$

$\omega_{c_2} = 8$

$\omega_h = 50.$

$\omega = \omega_1 = 0.5 \quad A = 20 \log\left(\frac{0.063}{j\omega}\right)$

$= 20 \log\left(\frac{0.063}{0.5}\right) = -18 \text{ db.}$

$\omega = \omega c_1 = 2 \; ; \quad A = 20 \log\left(\frac{0.063}{j\omega}\right)$

$= 20 \log\left(\frac{0.063}{2}\right) = -30.$

$\omega = \omega c_2 = 8 \; ; \; A = [\text{Slope from } \omega c_1 \text{ to } \omega c_2 \times \log\left(\frac{\omega c_2}{\omega c_1}\right)]$

$+ A \text{ (at } \omega = \omega c_1).$

$A = \left[-10 \times \log\left(\frac{8}{2}\right)\right] + (-30).$

$A = -54$

$\omega = \omega_h = 50, \; A = [\text{Slop from } \omega c_2 \text{ to } \omega_h \times \log\left(\frac{\omega_h}{\omega c_2}\right)] + A \text{ (at } \omega = \omega c_2)$

$A = \left[-60 \times \left(\frac{50}{8}\right)\right] + (-54)$

$A = -102.$

To find phase plot

$$G(j\omega) = \frac{e^{-0.2j\omega}}{j\omega(1+0.5j\omega)(1+0125\,j\omega).}$$

$$\phi = -90 - \tan^{-1} 0.5\omega - \tan^{-1} 0.125\omega \times -0.2\,\omega\left(\frac{180}{\pi}\right)$$

$$\phi = -0.2\omega \times \left(\frac{180}{\pi}\right) - 90 - \tan^{-1} 0.5\omega - \tan^{-1} 0.125\,\omega.$$

ω	$-0.2\omega \times \left(\frac{180}{\pi}\right)$	$\tan^{-1}0.5\omega$	$\tan^{-1}0.125\omega$	ϕ
0.01	-0.1146	0.2865	0.0116	-90
0.1	-1145	2.8624	0.716	-95.
0.5	-5.729	14.0362	3.576	-113
1	-11.459	26.5651	7.1250	-135
2	-22.918	45.0000	14.036	-171
3	-34.37	56.3099.	20.5560	-201
4	-45.83	63.4349.	26.5651.	-226.

Magnitude Plot

0.5 = -18 db

2 = -30 db

8 = -54 db.

50 = -102 db.

i) Phase Magin in 45°

$y = 180^\circ + \phi gc.$

$45 - 180^\circ \pm \phi gc.$

$\phi gc = -135.$

$-24 + 24 = 0.$

$20 \log k = 24$

$k = 10^{24/20}$

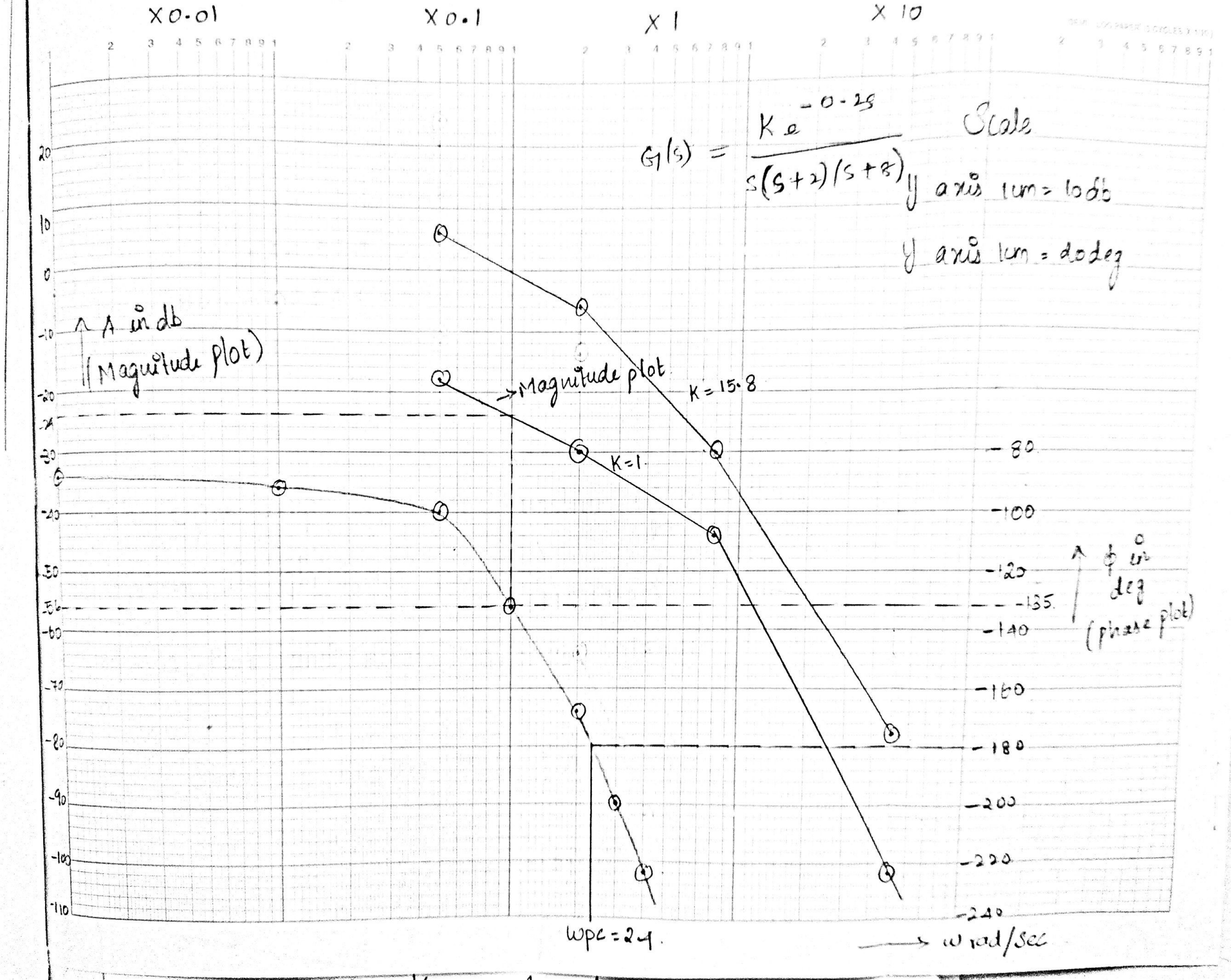
$k = 15.8.$

$0.5 = -18 + 24. = 6.0$

$2 = -30 + 24 = -6$

$8 = -54 + 24 = -30$

$50 = -102 + 24 = -78.$


X 0.01
X 0.1
X 1
X 10
$G(s) = \frac{K e^{-0.2s}}{s(s+2)(s+8)}$
Scale
y axis 1cm = 10db
y axis 1cm = 20deg
A in db
(Magnitude plot)
Magnitude plot
K = 15.8
K = 1
ϕ in deg
(phase plot)
$\omega_{pc} = 2.4$
ω rad/sec

Polar Plot

The open Loop Transfer Function of a unity feedback System is Given by $G(s) = \frac{1}{s(1+s)(1+2s)}$. Sketch the Polar Plot and determine phase Margin & gain Margin.

Soln:-

Step 1:- Replace s by $j\omega$

$$G(j\omega) = \frac{1}{j\omega(1+j\omega)(1+2j\omega)}$$

Step 2:- To find corner frequency.

$\omega_c = 1/1$ $\omega_{c_1} = 1/2$.

$\omega_{c_2} = \frac{1}{1}$ $\omega_{c_1} = 0.5$.

$\omega_{c_2} = 1$

Step 3 :- To find Magnitude value & phase value.

Magnitude value :-

$$|\sqrt{a^2+b^2}|$$

$$G(j\omega) = \frac{1}{j\omega(1+j\omega)(1+2j\omega)}$$

$(1+j\omega)$; $a=1$, $b=\omega$ $\Rightarrow \sqrt{1^2+\omega^2}$

$(1+2j\omega)$; $a=1$, $b=2\omega$ $\sqrt{1^2+(2\omega)^2}$

$$|G(j\omega)| = \frac{1}{\omega\sqrt{1^2+\omega^2}\ \sqrt{1^2+(2\omega)^2}}$$

Phase value :-

$$\angle G(j\omega) = \frac{1}{j\omega(1+j\omega)(1+j2\omega)}$$

$$\phi = -90 - \tan^{-1}\omega - \tan^{-1}2\omega$$

$\frac{1}{(1+j\omega)}$, $a=1$, $b=\omega$ $= -\tan^{-1} b/a = -\tan^{-1}\omega$

$\frac{1}{1+2j\omega}$, $a=1; b=2\omega$ $= -\tan^{-1}(b/a) = +\tan^{-1}\left(\frac{2\omega}{1}\right) = +\tan^{-1}2\omega$

Phase Margin

$\gamma = 180^\circ + \phi gc$

$\gamma = 180^\circ - 168.$

$\gamma = 12.$

Gain Margin.

$Kg = \frac{1}{G(j\omega pc)}$.

$= \frac{1}{0.7}$.

$= 1.42.$

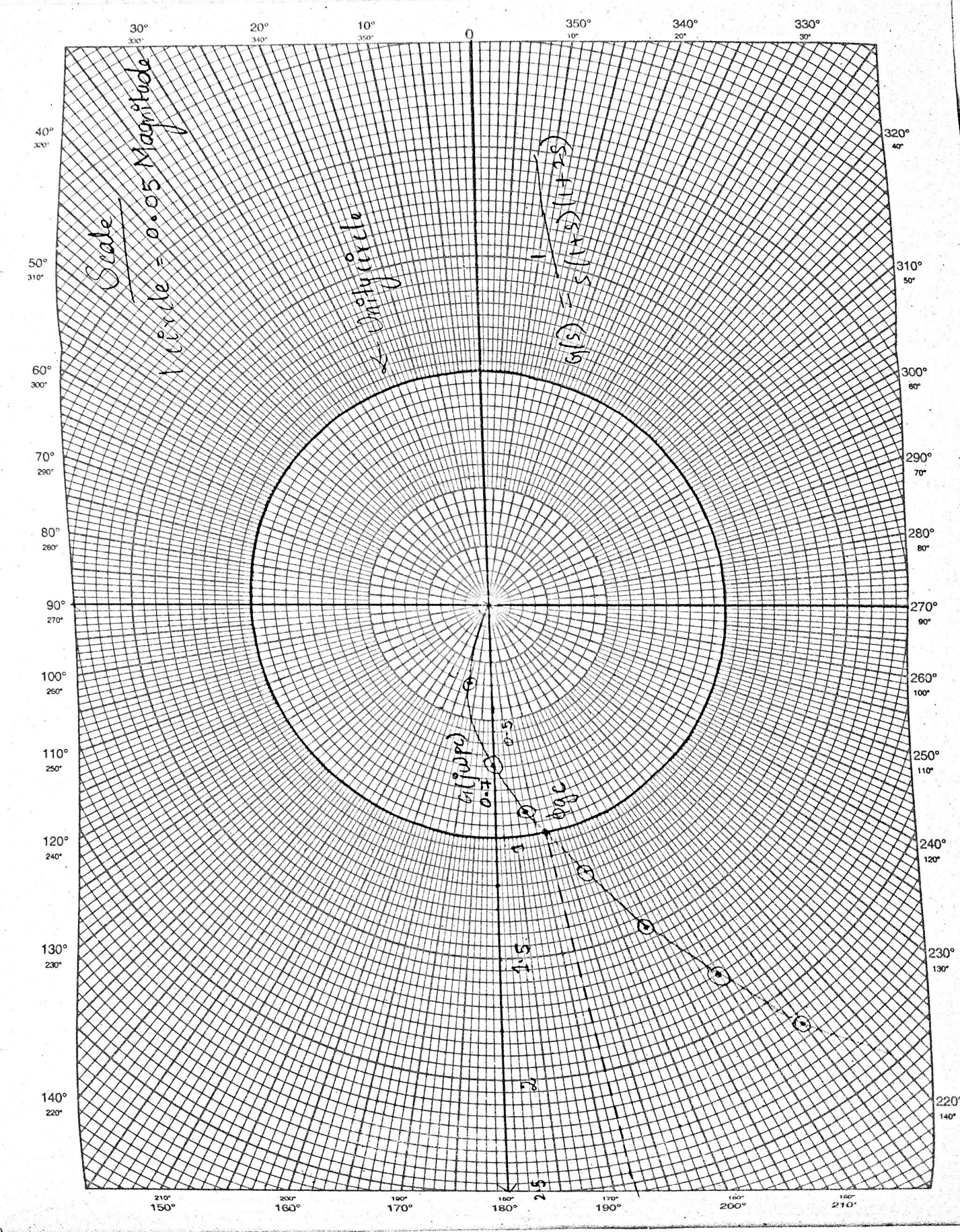
$$|G(j\omega)| = \frac{1}{\omega\sqrt{1^2+\omega^2}\sqrt{1^2+(2\omega)^2}}$$

$$= \frac{1}{\omega\sqrt{(1+\omega^2)(1+4\omega^2)}} = \frac{1}{\omega\sqrt{1+4\omega^2+\omega^2+4\omega^4}}$$

$$|G(j\omega)| = \frac{1}{\omega\sqrt{1+5\omega^2+4\omega^4}} \leftarrow \text{Magnitude}$$

$$\angle G(j\omega) = -90 - \tan^{-1}\omega - \tan^{-1}2\omega \leftarrow \text{phase}$$

ω rad/sec	0.3	0.35	0.4	0.45	0.5	0.6	0.7	1
$\lvert G(j\omega)\rvert$	2.7	2.2	1.81	1.50	~~0.0039~~ 1.2	~~0.0023~~ 0.9	0.0014 0.7	~~0.3162~~ 0.3
$\angle G(j\omega)$	-138	-145	-150	-156	~~-253~~ -162	~~-256~~ -171	-180	-198


Scale
1 circle = 0.05 Magnitude
Unit circle
$G_1(s) = \frac{1}{s(1+s)(1+2s)}$
$G_1(j\omega)$
0.4
0.5
0.6
1
1.5
2
2.5
30° 330°
20° 340°
10° 350°
0
350° 10°
340° 20°
330° 30°
40° 320°
50° 310°
60° 300°
70° 290°
80° 280°
90° 270°
100° 260°
110° 250°
120° 240°
130° 230°
140° 220°
150° 210°
160° 200°
170° 190°
180°

2) Draw the ptdar plot for the given transfer function

$$G(s) = \frac{K}{s(1+0.5s)(1+As)}$$ determine the value of

K, so that (i) gain Margin is 20db (ii) phase Margin is 30°

Soln :-

$s = j\omega \quad K = 1.$

$$\omega c_2 = \frac{1}{0.5} = 2$$

$$G(j\omega) = \frac{1}{j\omega(1+0.5j\omega)(1+4j\omega)}$$

$$\omega c_2 = 2.$$

$$|G(j\omega)| = \frac{1}{\omega\sqrt{1+(0.5)^2}\sqrt{1+(4\omega)^2}}$$

$\omega_1 = \frac{1}{4} = 0.25$

$\omega_1 = 0.25$

$$\angle G(j\omega) = -90 - \tan^{-1} 0.5\omega - \tan^{-1} 4\omega.$$

ω rad/sec	0.3	0.4	0.5	0.6	0.8	1	1.2
$\|G(j\omega)\|$	2.71	2.2	0.87	0.51	0.35	0.22	0.15
$\angle G(j\omega)$	-149	-159	-167	-174	-184	-193	-199

From the polar plot

$K = 1.$

Gain Margin $kg = \frac{1}{0.44}$

$\boxed{Kg = 2.27}$

Gain Margin

$20 \log 2.27 = 7.12 db$

Phase Margin (γ)

$\gamma = 180° + \phi gc.$

$\gamma = 180 - 163$

$\boxed{\gamma = 17°}$

Case (i)

Gain Margin is 20db

$K = \frac{GA}{GB}$

$\boxed{20 \log \frac{1}{GA} = 20}$

$\log \frac{1}{GA} = \frac{20}{20} = 1$

$\frac{1}{GA} = 10^1$

$GA = \frac{1}{10} = 0.1$

The value of K is given by

$K = \frac{GA}{GB} = \frac{0.1}{0.44}.$

$\boxed{K = 0.227.}$

case (ii) $\boxed{K = \frac{GD}{Gc}}$

Phase Margin is 30°

$\gamma = 180° + \phi gc_2$

$\phi gc_2 = 30 - 180$

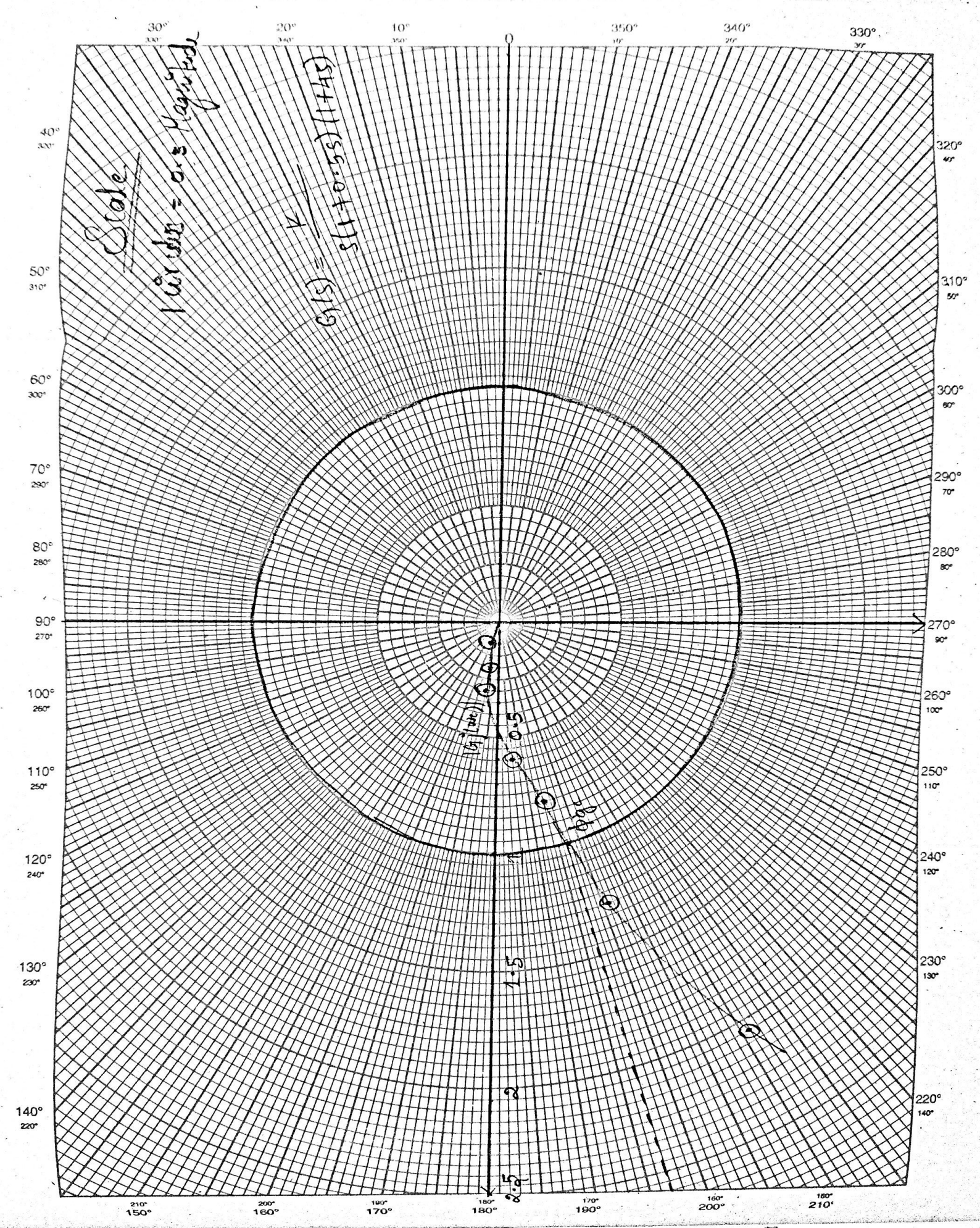
$\boxed{\phi gc = -150}$

-150° cut in polar plot is 2.04.

$Gc = 2.04$; $GD = 1$

$K = \frac{GD}{Gc} = \frac{1}{2.04}$

$\boxed{K = 0.49.}$


Scale
1 unit = 0.5 kg/cm²
G(S) = K / S(1+0.5S)(1+2S)
30°
20°
10°
0
350°
340°
330°
40°
320°
50°
310°
60°
300°
70°
290°
80°
280°
90°
270°
100°
260°
110°
250°
120°
240°
130°
230°
140°
220°
210°
200°
190°
180°
170°
160°
150°
0.5
1.0
1.5
2.0
2.5
bbo

3) Draw the polar plot. To find phase Margin & Gain Margin

$$G(s) = \frac{1}{s^2(1+s)(1+2s)}$$

$$s = j\omega$$

$$G(j\omega) = \frac{1}{(j\omega)^2(1+j\omega)(1+2j\omega)}$$

$$\omega_{c_2} = \frac{1}{1} = 1$$

$$\omega_{c_1} = \frac{1}{2} = 0.5.$$

$$|G(j\omega)| = \frac{1}{\omega^2\sqrt{1+\omega^2}\sqrt{1+(2\omega)^2}}$$

$$= \frac{1}{\omega^2\sqrt{1+\omega^2(1+4\omega^2)}}$$

$$|G(j\omega)| = \frac{1}{\omega^2\sqrt{1+5\omega^2+4\omega^4}} \leftarrow \text{Magnitude}$$

$$\angle G(j\omega) = -180 - \tan^{-1}\omega - \tan^{-1}2\omega \leftarrow \text{phase}$$

ω rad/sec	0.45	0.5	0.55	0.6	0.65	0.7	0.75	1.
$\lvert G(j\omega)\rvert$	3.3	2.5	1.9	1.5	1.2	0.9	0.8	0.3
$\angle G(j\omega)$	-246	-251	-256	-261	-265	-269	-273	-288

Phase Margin (γ):-

$$\gamma = 180^\circ + (-270^\circ)$$

$$\gamma = -90.$$

Gain Margin (Kg).

$$K_g = \frac{1}{|G(j\omega_{pc})|}$$

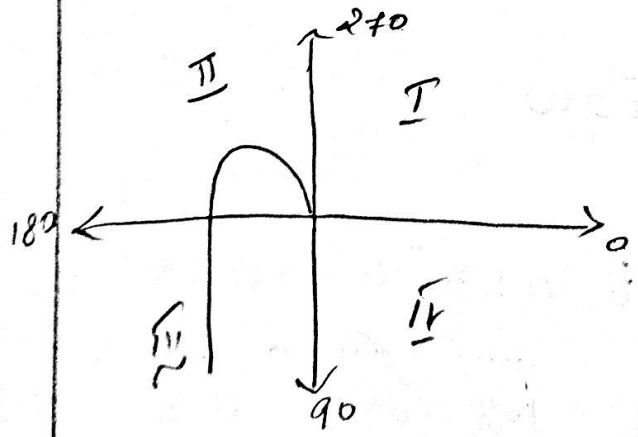
$$= \frac{1}{0}$$

$$K_g = \infty.$$

Reference:-

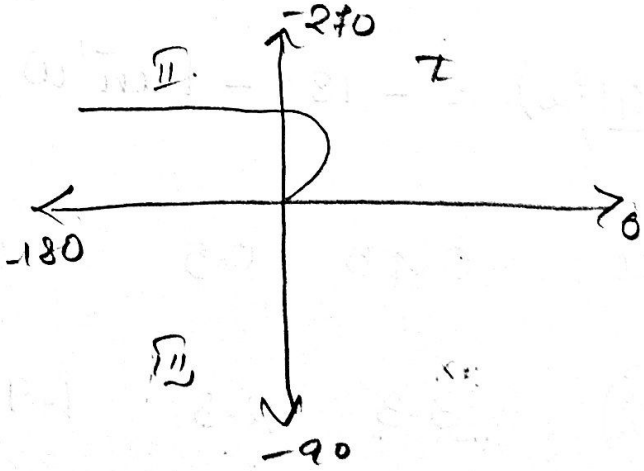
$$G(s) = \frac{1}{s(1+sT_1)(1+sT_2)}$$

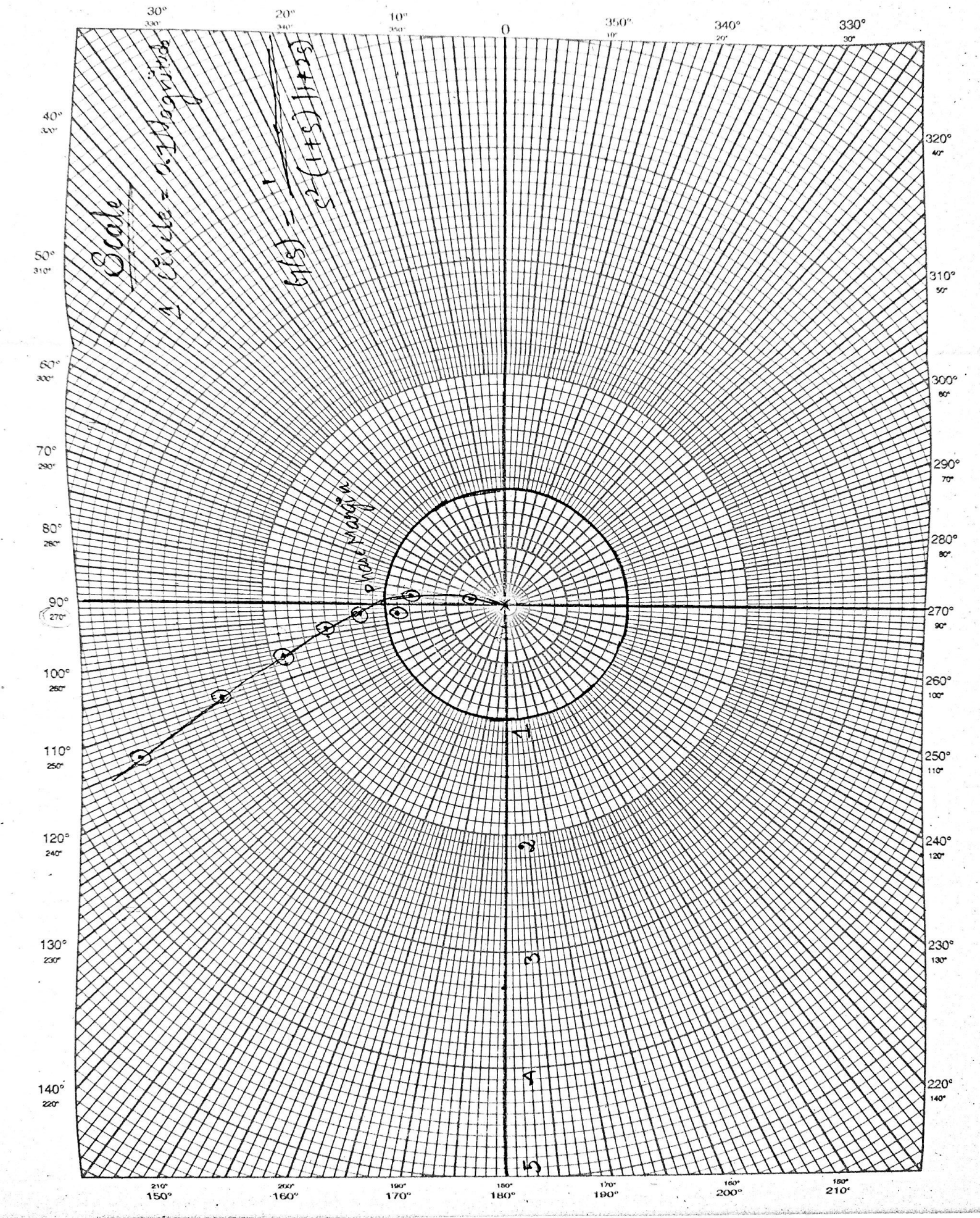
Type: 1 order: 3.



$$G(s) = \frac{1}{s^2(1+sT_1)(1+sT_2)}$$

Type: 2 order: 4.




Scale
1 circle = 2 Rayleigh
$\psi(\theta,s) = \frac{1}{s^2(1+s^2)^{1/2}}$
plane wave
0
10°
20°
30°
340°
350°
330°
320°
310°
300°
290°
280°
270°
260°
250°
240°
230°
220°
210°
200°
190°
180°
170°
160°
150°
140°
130°
120°
110°
100°
90°
80°
70°
60°
50°
40°
1
2
3
4
5

UNIT - IV

Stability Analysis.

The closed Loop Transfer function M(S) can be expressed as two polynomials closed Loop Transfer Function is called characteristic Equations.

The Roots of characteristic Equation are poles of Transfer Function.

$$M(S) = \frac{(S+z_1)(S+z_2)+\ldots\ldots(S+z_m)}{(S+P_1)(S+P_2)+\ldots\ldots(S+P_m)}$$

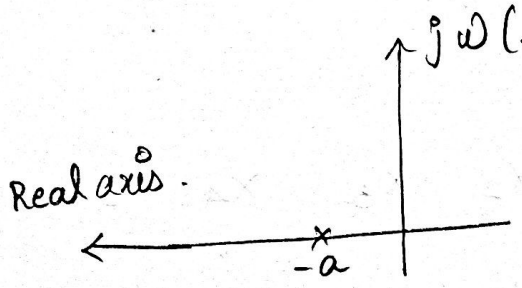
$z_1, z_2 \ldots z_m \rightarrow$ zeros

$P_1, P_2 \ldots P_m \rightarrow$ poles

The poles May be at origin (or) May be Imaginary (or) May be Left half S-plane (or) May be Right half S-plane.

Transfer Function ; M(S) & location of

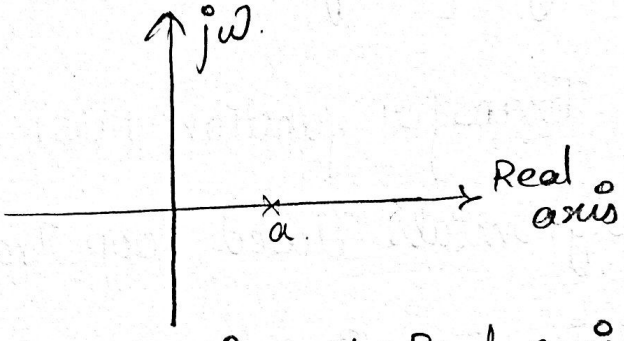
i) $M(S) = \frac{A}{S+a}$; $S = -a$.



System is stable

Roots are in -Ve Real axis

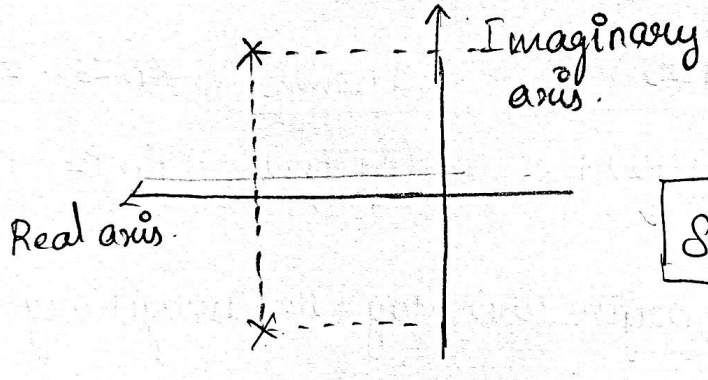
ii) $M(s) = \frac{A}{s-a}$; $s = +a$.



System is unstable

Roots are in +Ve Real axis.

iii) $M(s) = \frac{A}{s+a+jb} + \frac{A}{s+a-jb}$.



System is Marginally stable.

Roots are in complex conjugate.

Routh Hurwitz Criterion :-

Consider two consecutive Rows of Routh array as shown below.

Given $\begin{cases} s^n : & x_0 & x_1 & x_2 & x_3 & x_4 & x_5 \\ s^{n-1} : & y_0 & y_1 & y_2 & y_3 & y_4 & y_5 \end{cases}$

To find $\rightarrow s^{n-2}$: $\begin{matrix} ? & ? & ? & ? & ? & ? \\ z_0 & z_1 & z_2 & z_3 & z_4 & z_5 \end{matrix}$.

$$z_0 = \begin{vmatrix} x_0 & x_1 \\ y_0 & y_1 \end{vmatrix} = \frac{y_0 x_1 - y_1 x_0}{y_0}$$

$$z_1 = \begin{vmatrix} x_0 & x_2 \\ y_0 & y_2 \end{vmatrix} = \frac{y_0 x_2 - y_2 x_0}{y_0}$$

$$z_2 = \begin{vmatrix} x_0 & x_3 \\ y_0 & y_3 \end{vmatrix} = \frac{y_0 x_3 - y_3 x_0}{y_0}$$

$$z_3 = \begin{vmatrix} x_0 & x_4 \\ y_0 & y_4 \end{vmatrix} = \frac{y_0 x_4 - y_4 x_0}{y_0}$$

$$z_4 = \begin{vmatrix} x_0 & x_5 \\ y_0 & y_5 \end{vmatrix} = \frac{y_0 x_5 - y_5 x_0}{y_0}$$

Types of cases in Routh array :-

case(i): Normal Routh array.

The Non zero element in the first column and there is no sign change than all the Roots Lying in Left half S-plane, then the System is Stable.

If there is sign change in first column then the System is Unstable.

Case (ii) : A Row of all zeros.

In The Routh Hurwitz a Row of all element zeros means determine the auxilary polynomial with Respect to s and get $\frac{dA}{ds}$

The Rows of zeros is Replaced with co-efficient $\frac{dA(s)}{ds}$

Case (iii) :- First element of Row is zero.

zero is Encountered with first element of a Row then all the Element of Next will be Infinite.

To overcome this problem Let $0 \rightarrow \varepsilon$ and complete the array in case I.

Finally $\varepsilon \rightarrow 0$ and determine the value of Elements of array in case II.

1) Using Routh-Hurwitz criterion determine the stability of the System Represented the characteristic Equation.

$$s^4 + 8s^3 + 18s^2 + 16s + 5 = 0$$

Comment the Location of Roots of characteristic Equation.

Soln:-

s^4	1	18	5	s^4	1	18	5
s^3	$\frac{8}{8}$	$\frac{16}{8}$		s^3	1	2	

s^4	1	18	5
s^3	1	2	-
s^2	16	5	-
s^1	1.9	-	-
s^0	5	-	-

$$s21 = \frac{1 \times 18 - 2 \times 1}{1} = \frac{18-2}{1} = 16$$

$$s^{22} = \frac{1 \times 5 - 0}{1} = 5.$$

$$s^{11} = \frac{16 \times 2 - 5 \times 1}{16} = 1.9.$$

Result:-

* In the first there is no zero & No Sign change
* System is Stable.
* Therefore Four roots lying in left half S-plane.

2) Construct the Routh array and determine the Stability of the System whose characteristic Equation are

$$s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0.$$

also determine whether the roots are lying in Right half (or) Left half (or) Imaginary axis.

Soln :-

$$s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0.$$

s^6	1	8	20	16.
s^5	1	6	8	÷

$(\div 2)$.

s^6	1	8	20	16.
s^5	1	6	8	–
s^4	2	12	16	–

$$s^{41} = \frac{1\times 8 - 6\times 1}{1} = \frac{8-6}{1}$$

$(\div 2)$ $s^{41} = 2$

$$s^{42} = \frac{1\times 20 - 8\times 1}{1}$$

$$s^{42} = 20 - 8.$$

$$s^{42} = 12.$$

s^6	1	8	20	16.
s^5	1	6	8	–
s^4	1	6	8	–
s^3	0	0	–	–

$$s^{31} = \frac{1\times 6 - 6\times 1}{1} = 0$$

$$s^{32} = \frac{1\times 8 - 8\times 1}{1} = 0$$

Case (ii) row of all zero.

$$A = s^4 + 6s^2 + 8 = 0.$$

$$\frac{dA}{ds} = 4s^3 + 12s.$$

$$4 : 12. \quad (\div 4)$$

1 : 3.

S^6	1	8	20	16.
S^5	1	6	8	–
S^4	1	6	8	–
S^3	1	3	–	–
S^2	3	8	–	–
S^1	0.33	–	–	–
S^0	8	–	–	–

$S^4 + 6S^2 + 8 = 0.$

$a = 1 ; \quad b = 6 ; \quad c = 8$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{36 - 4(1)(8)}}{2}$$

$$= \frac{-6 \pm \sqrt{36 - 32}}{2} = \frac{-6 \pm \sqrt{4}}{2} = \frac{-6 \pm 2}{2}.$$

$= -3 \pm 1.$

$x = -3 \pm 1$

$x = -3 + 1 \qquad x = -3 - 1$

$x = -2 \qquad x = -4$

$S^2 = x \qquad S = \pm\sqrt{x}$

$s = \pm\sqrt{-2}$; $s = \pm\sqrt{-4}$

$s = \pm j\sqrt{2}$; $s = \pm j\sqrt{4}$.

$s = +j\sqrt{2}$; $s = -j\sqrt{2}$; $s = +j\sqrt{4}$; $s = -j\sqrt{4}$.

Result :-

System is Marginally Stable.

Four Roots lying in Imaginary axis and Remaining two roots in Left Side.

3) Construct the Routh-array

$$s^7 + 9s^6 + 24s^5 + 24s^4 + 24s^3 + 24s^2 + 23s + 15 = 0$$

Determine the Location of Roots :

Soln :-

$$s^7 + 9s^6 + 24s^5 + 24s^4 + 24s^3 + 24s^2 + 23s + 15 = 0.$$

s^7	1	24	24	23	
s^6	9	24	24	15.	$\div 3$.
s^7	1	24	24	23	
s^6	3	8	8	5.	

s^7	1	24	24	23
s^6	3	8	8	5.
s^5	1	1	1	(÷) 21
s^4	5	5	5	(÷ 5)
s^3	0	0	–	– –

s^7	1	24	24	23
s^6	3	8	8	5.
s^5	1	1	1	–
s^4	1	1	1	–
s^3	0	0	0	–

$$A = s^4 + s^2 + 1 = 0.$$

$$\frac{dA}{ds} = 4s^3 + 2s.$$

4 : 2

2 : 1.

s^7	1	24	24	23.
s^6	3	8	8	5.
s^5	1	1	1	–

S^4	1	1	1	–
S^3	2	1	–	–
S^2	0.5	2	–	–
S^1	-7	–	–	–
S^0	1	–	–	–

$$S^4 + S^2 + 1 = 0.$$

$$S^2 = x.$$

$$x^2 + x + = 0.$$

$$a = 1; \quad b = 1; \quad c = 1.$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1-4}}{2} \Rightarrow \frac{-1 \pm \sqrt{-3}}{2}$$

$$= \frac{-1}{2} \pm \frac{j\sqrt{3}}{2}$$

$$= -0.5 \pm j0.86.$$

$$S^2 = x.$$

$$S = \pm\sqrt{x}$$

$$S = \pm\sqrt{-0.5 + j0.86}$$

$$S = \pm\sqrt{-0.5 - j0.86}$$

$$S = +\sqrt{-0.5+j0.86},\ -\sqrt{-0.5+j0.86},\ +\sqrt{-0.5-j0.86},$$

$$-\sqrt{-0.5-j0.86}$$

Result :-

System is Marginally stable.

Four roots in Imaginary axis.

Three roots in left side.

4) Construct Routh Hurwitz & comment the roots.

$$S^5+S^4+2S^3+2S^2+3S+5=0.$$

S^5	1	2	3
S^4	1	2	5
S^3	0	-2	-

(case (iii) A row of one element is zero.

S^5	1	2	3
S^4	1	2	5
S^3	ε	-2	-
S^2	$\frac{2\varepsilon+2}{\varepsilon}$	5	-
S^1	$\frac{-(4\varepsilon+4+5\varepsilon^2)}{2\varepsilon+2}$	-	-
S^0	5	-	-

$$S^{21} = \frac{2\varepsilon-(-2)}{\varepsilon} = \frac{2\varepsilon+2}{\varepsilon}.$$

$$S^{11} = \frac{\frac{2\varepsilon+2}{\varepsilon}(-2)-5\varepsilon}{\frac{2\varepsilon+2}{\varepsilon}} = \frac{\frac{-4\varepsilon-4}{\varepsilon}-5\varepsilon}{\frac{2\varepsilon+2}{\varepsilon}}$$

$$S^{11} = -\frac{4\varepsilon-4-5\varepsilon^2}{2\varepsilon+2}$$

$$S^{11} = -\frac{(4\varepsilon+4+5\varepsilon^2)}{2\varepsilon+2}$$

From the above table $\varepsilon \to 0$.

S^5	1	2	3.
S^4	1	2	5.
S^3	0	-2	-
S^2 ↓+ ↓-	∞	5	-
S^1 ↓-	-2	-	-.
S^0 ↓+	5.	-	-.

Result:-

* System is Unstable.
* Two roots lying in right side. three roots lying in Left side.

5) Determine the Range of K for the Stability Unity feedback system whose open loop Transfer function is

$$G(S) = \frac{K}{S(S+1)(S+2)}.$$

Soln:-

$[\because H(S) = 1]$

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1 + G(S)H(S)}.$$

$$= \frac{\frac{K}{S(S+1)(S+2)}}{1 + \frac{K}{S(S+1)(S+2)} \cdot 1} = \frac{\frac{K}{S(S+1)(S+2)}}{\frac{S(S+1)(S+2)+K}{S(S+1)(S+2)}}$$

$$= \frac{K}{S(S+1)(S+2)+K}$$

Root Locus :-

Root Locus is a technique & powerful tool for adjusting the Location of closed Loop poles to achieve the desired system performance by varying one (or) More system parameter.

closed Loop Transfer function of system.

$$G(S) = \frac{K}{S(S+P_1)(S+P_2)} \begin{matrix} \rightarrow \text{zeros} \\ \rightarrow \text{poles.} \end{matrix}$$

Procedure for constructing Root Locus:

Step 1:- Location of poles & zeros:

Draw the Real & Imaginary axis in the ordinary graph sheet and choose same scale of both Real and Imaginary axis.

The poles are Marked in 'x' & zeros are Marked 'o'

Step 2:- Root Location of Real axis:

If Number of Poles & zeros in odd Number, then the test point Lies on Real axis.

If Number of poles & zeros in even Number, then the test point does not Lies on Real axis.

Step 3:- Angle of asymptotes & Centroid:

$$\text{Asymptotes} = \frac{\pm 180(2q+1)}{N-m}$$

[N = No. of poles
M = No. of zeros]

$$\text{Centroid} = \frac{\text{Sum of poles} - \text{Sum of zeros}}{N-m.}$$

Step 4:- Break away & Break in point:

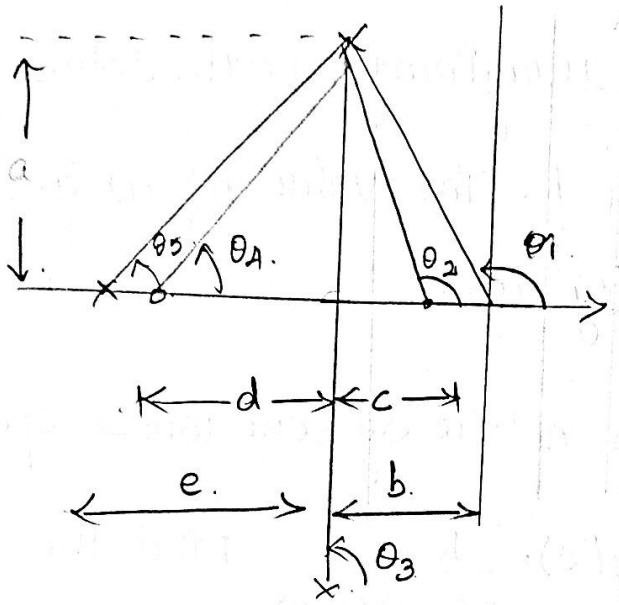
If the Root Locus 2- poles on Real axis there is Break away point.

If the Root Locus 2- zeros on Real axis there is Break in point.

Step 5:- Angle of departure & Angle of arrival:

If the Root Locus have complex poles there is a angle of departure.

Angle of departure = 180° - (sum of poles) + (sum of zeros).



$\theta_1 = 180^\circ - \tan^{-1}\frac{a}{b}$

$\theta_2 = 180 - \tan^{-1}\frac{a}{c}$

$\theta_3 = 90^\circ$

$\theta_4 = \tan^{-1}\frac{a}{d}$

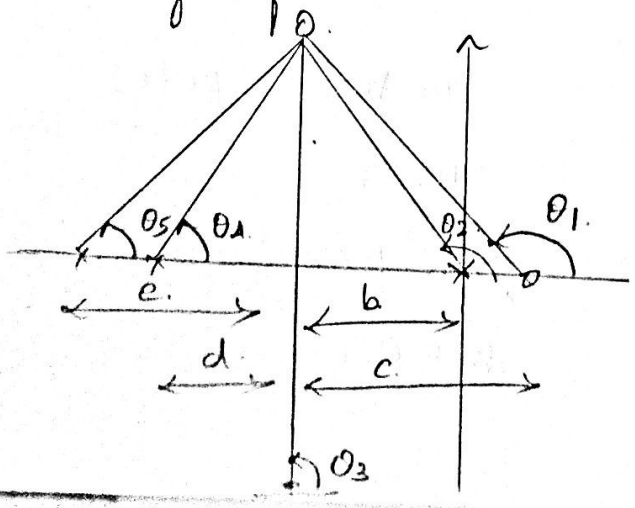
$\theta_5 = \tan^{-1}\frac{a}{e}$

Angle of depature = $180^\circ - (\theta_1 + \theta_3 + \theta_5) + (\theta_2 + \theta_4)$.

Angle of arrival

If the Root Locus have complex zero there is a angle of arrival.

Angle of arrival = 180° - (Sum of zeros) + (sum of poles).



$\theta_1 = 180^\circ - \tan^{-1}\frac{a}{c}$

$\theta_2 = 180^\circ - \tan^{-1}\frac{a}{b}$

$\theta_3 = 90^\circ$

$\theta_4 = \tan^{-1}\frac{a}{d}$

$\theta_5 = \tan^{-1}\frac{a}{e}$

Angle of arrival $= 180^\circ - (\theta_1 + \theta_3) + (\theta_2 + \theta_4 + \theta_5)$.

Step 6:- Point of Intersection of Root Locus with Imaginary axis.

Let $s = j\omega$ in the characteristic Equation and Separate the Real & Imaginary part. Solve the two Equation to find ω & K. The value of ω gives the crossing the Imaginary axis.

) Sketch the Root Locus of the System whose open Loop. Transfer function is $G(s) = \frac{K}{s(s+1)(s+2)}$ Find the value of K. So that damping Ratio of the System is 0.5.

Soln:-

Step 1:- Location of poles & zeros.

$$G(s) = \frac{K}{s(s+1)(s+2)} \begin{matrix} \rightarrow \text{zeros} \\ \rightarrow \text{poles} \end{matrix}$$

$s = 0, s = -1, s = -2.$

$P_1 = 0, P_2 = -1, P_3 = -2.$

n = no. of poles

$n = 3.$

m = no. of zeros.

$m = 0.$

Step 2 :- Root Locus on Real axis.

Step 3 :- Angle of Asymptotes.

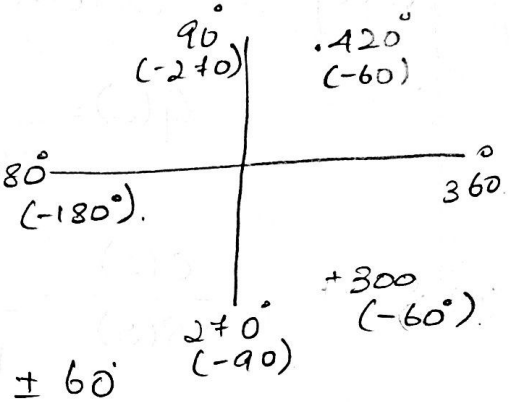
$$\text{Asymptotes} = \frac{\pm 180^\circ(2q+1)}{n-m} \qquad q = 0, 1, 2, 3$$

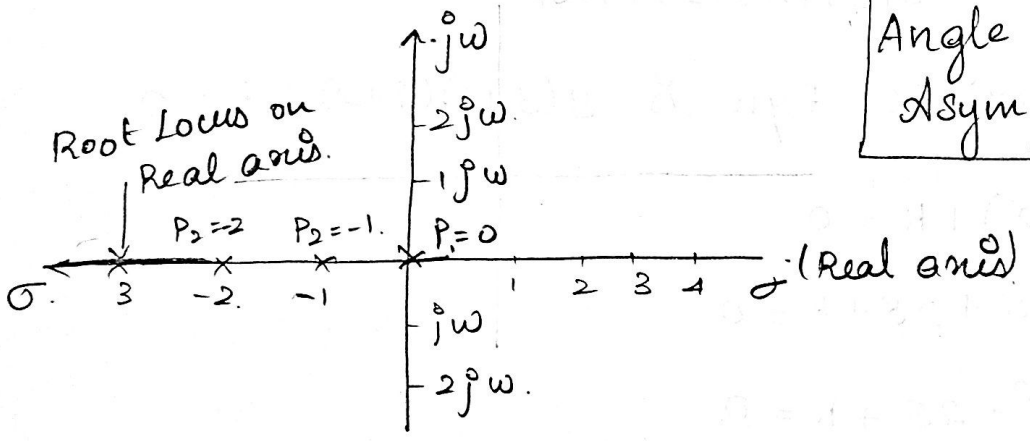
$$q = 0, \quad = \frac{\pm 180^\circ(2(0)+1)}{3-0} = \pm 60.$$

$$q = 1 = \frac{\pm 180^\circ(2(1)+1)}{3} = \pm 180^\circ$$

$$q = 2 = \frac{\pm 180^\circ(2(2)+1)}{3} = \pm 300 = \pm 60^\circ$$

$$q = 3 = \frac{\pm 180^\circ(2(3)+1)}{3} = \pm 420^\circ = \pm 60^\circ.$$





Angle of Asymptotes $= \pm 60^\circ$.

$$\text{Centroid} = \frac{\text{Sum of poles} - \text{Sum of zeros}}{n-m} = \frac{0-1-2-0}{3-0}$$

$$= \frac{-3}{3}$$

Centroid $= -1$.

(4) { Break away point = two poles on Real axis.
{ Break in point = two zeros on Real axis.

(3) Angle of departure = complex poles.

Angle of arrival = complex poles.

Step 4:- Break away point.

$$G(S) = \frac{K}{S(S+1)(S+2)}$$

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)H(S)} = \frac{\frac{K}{S(S+1)(S+2)}}{1+\frac{K}{S(S+1)(S+2)}}$$

$$\frac{C(S)}{R(S)} = \frac{K}{S(S+1)(S+2)+K}$$

The characteristic Eqn is $S(S+1)(S+2)+K=0$.

$$(S^2+S)(S+2)+K=0$$

$$S^3+2S^2+S^2+2S+K=0$$

$$S^3+3S^2+2S+K=0$$

$$K = -S^3-3S^2-2S$$

$$K = -[S^3+3S^2+2S]$$

$$\frac{dK}{dS} = 0$$

$$\frac{dK}{dS} = -[3S^2+6S+2]$$

$$a=3,\ b=6,\ c=2.$$

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{6^2 - 4(3)(2)}}{2(1)}$$

$$s = -4.73$$

$$s = -1.26$$

$$K = -[s^3 + 3s^2 + 2s]$$

$$= -[(-4.73)^3 + 3(-4.73)^2 + 2(-4.73)]$$

$$K = 48.16$$

$$K = -[s^3 + 3s^2 + 2s]$$

$$= -[(-1.26)^3 + 3(-1.26)^2 + 2(-1.26)]$$

$$K = -0.24$$

Break away point = -4.73 (K is +ve).

Step 5 :- Angle of depature & Angle of Arrival :

There is no complex poles & zeros, then there is no possibility of angle of depature & Angle of arrival.

Step 6 :- Crossing point on Imaginary axis :

$$s^3 + 3s^2 + 2s + K = 0.$$

$$s = j\omega.$$

$$(j\omega)^3 + 3(j\omega)^2 + 2(j\omega) + K = 0.$$

$$j^3\omega^3 + 3j^2\omega^2 + 2j\omega + K = 0.$$

$$-j\omega^3 - 3\omega^2 + 2j\omega + K = 0 \qquad [j^2 = -1].$$

Equate Imaginary

$$-j\omega^3 + 2j\omega = 0.$$

$$-j\omega^3 = -2j\omega.$$

$$\omega^3 = 2\omega$$

$$\omega^2 = 2.$$

$$\omega = \pm\sqrt{2}.$$

$$\omega = \pm j1.41.$$

Equate Real Value.

$$-3\omega^2 + K = 0.$$

$$K = 3\omega^2. \qquad [\omega^2 = 2].$$

$$K = 3 \times 2.$$

$$K = 6.$$

Unit-5

State Variable Analysis

Controllability and observability:

1. check the Controllability and Observability of the system State space Representation is given as.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \underbrace{\begin{bmatrix}0 & 0 & 1\\ -2 & -3 & 0\\ 0 & 2 & -3\end{bmatrix}}_{A\ 3\times3}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \underbrace{\begin{bmatrix}0\\ 2\\ 0\end{bmatrix}}_{B\ 3\times1} u \; ; \quad y = \underbrace{\begin{bmatrix}1 & 0 & 0\end{bmatrix}}_{C\ 1\times3}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

Sol:-

$$\dot{x}(t) = Ax(t) + Bu(t).$$

$$y(t) = Cx(t) + Du(t).$$

$$A = \begin{bmatrix}0 & 0 & 1\\ -2 & -3 & 0\\ 0 & 2 & -3\end{bmatrix}; \quad B = \begin{bmatrix}0\\ 2\\ 0\end{bmatrix}; \quad C = \begin{bmatrix}1 & 0 & 0\end{bmatrix}$$

Controllability formula:-

$$Q_c = \begin{bmatrix}B & AB & A^2B\end{bmatrix};$$

Observability formula:-

$$Q_o = \begin{bmatrix}C^T & A^TC^T & (A^T)^2C^T\end{bmatrix}$$

$$A^2 = A \cdot A = \begin{bmatrix} 0 & 0 & 1 \\ -2 & -3 & 0 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ -2 & -3 & 0 \\ 0 & 2 & -3 \end{bmatrix}$$

$$\boxed{A^2 = \begin{bmatrix} 0 & 2 & -3 \\ 6 & 9 & -2 \\ -4 & -12 & 9 \end{bmatrix}} ;$$

$$A \cdot B = \begin{bmatrix} 0 & 0 & 1 \\ -2 & -3 & 0 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$\boxed{AB = \begin{bmatrix} 0 \\ -6 \\ 4 \end{bmatrix}} \quad ; \quad A^2B = \begin{bmatrix} 0 & 2 & -3 \\ 6 & 9 & -2 \\ -4 & -12 & 9 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$\boxed{A^2B = \begin{bmatrix} 4 \\ 18 \\ -24 \end{bmatrix}}$$

$$\boxed{Q_c = \begin{bmatrix} C^T & A^T C^T & (A^T)^2 C^T \end{bmatrix}}$$

$$A^T = \begin{bmatrix} 0 & 0 & 1 \\ -2 & -3 & 0 \\ 0 & 2 & -3 \end{bmatrix}^T = \begin{bmatrix} 0 & -2 & 0 \\ 0 & -3 & 2 \\ 1 & 0 & -3 \end{bmatrix}$$

$$C^T = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$(A^T)^2 = \begin{bmatrix} 0 & -2 & 0 \\ 0 & -3 & 2 \\ 1 & 0 & -3 \end{bmatrix} \begin{bmatrix} 0 & -2 & 0 \\ 0 & -3 & 2 \\ 1 & 0 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 6 & -4 \\ 2 & 9 & -12 \\ -3 & -2 & 9 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} 0 & -2 & 0 \\ 0 & -3 & 2 \\ 1 & 0 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad ; \quad (A^T)^2 C^T = \begin{bmatrix} 0 & 6 & -4 \\ 2 & 9 & -12 \\ -3 & -2 & 9 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{(A^T)^2 C^T = \begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix}}$$

$$Q_o = \begin{bmatrix} C^T & A^T C^T & (A^T)^2 C^T \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 1 & -3 \end{bmatrix}$$

$|Q_o| = 1 \times -2 = -2 \neq 0.$

Hence the System is Completely observable.

$$Q_c = \begin{bmatrix} B & AB & A^2B \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 4 \\ 2 & -6 & 18 \\ 0 & 4 & -24 \end{bmatrix}$$

$|Q_c| = 4 \times 8 = 32. \neq 0.$

Hence the System is Completely ~~observable~~. Controllable.

② The State Equation and Initial Condition Vector of an Linear-time-Invariant System are given below. Determine Solution of State Equation.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}; \qquad x_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Sol:-

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\boxed{e^{At} = L^{-1}\left[(SI - A)^{-1}\right]}$$

$$\boxed{(SI - A)^{-1} = \frac{\left[\text{Co-factor of } (SI-A)\right]^T}{\text{Determiant of } (SI-A)}}$$

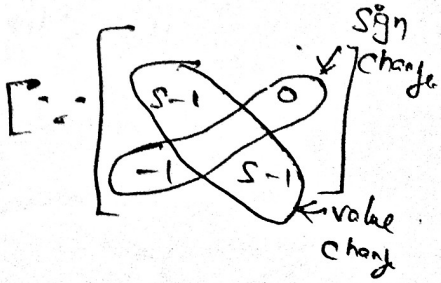
$$SI - A = S\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\boxed{SI - A = \begin{bmatrix} S-1 & 0 \\ -1 & S-1 \end{bmatrix}}$$

$$\boxed{|SI - A| = (S-1)^2}$$

$$\text{Co-factor of } (SI-A) = \begin{bmatrix} S-1 & 0 \\ 1 & S-1 \end{bmatrix}$$



$$(SI-A)^{-1} = \frac{1}{(s-1)^2}\begin{bmatrix} s-1 & 0 \\ 1 & s-1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s-1}{(s-1)^2} & 0 \\ \frac{1}{(s-1)^2} & \frac{s-1}{(s-1)^2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s-1} & 0 \\ \frac{1}{(s-1)^2} & \frac{1}{s-1} \end{bmatrix}$$

$$\boxed{e^{At} = L^{-1}(SI-A)^{-1}}$$

$$= L^{-1}\begin{bmatrix} \frac{1}{s-1} & 0 \\ \frac{1}{(s-1)^2} & \frac{1}{s-1} \end{bmatrix}$$

$$\boxed{e^{At} = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}}$$

$X(t) = e^{At}X_0 + L^{-1}\phi(s)BU(s)$

Solution of the State Equation is

$$X(t) = e^{At}X_0$$

$$= \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\boxed{X(t) = \begin{bmatrix} e^t \\ te^t \end{bmatrix}}$$

(3) Construct a state Model of a system characterized by the differential Equation.

$$\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 11\frac{dy}{dt} + 6y + u = 0. \quad \rightarrow (1)$$

Sol:-

$$\dot{x}_3 = \frac{d^3y}{dt^2}. \quad ; \qquad \dot{x}_1 = \frac{dy}{dt} \qquad \dot{x}_2 = \frac{d^2y}{dt}.$$

$$\left.\begin{aligned} x_1 &= y \\ x_2 &= dy/dt \\ x_3 &= \frac{d^2y}{dt^2} \\ \dot{x}_3 &= ? \end{aligned}\right\} \rightarrow (2)$$

Sub (2) in (1)

we get $\dot{x}_3$:-

$$\dot{x}_3 + 6x_3 + 11x_2 + 6x_1 + u = 0.$$

[∵ we know that Pre-define value.

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\boxed{\dot{x}_3 = -6x_3 - 11x_2 - 6x_1 - u.}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} u \quad \text{Eq.}$$

$$\dot{x} = Ax + Bu \quad \text{Eq. (4)}$$

④ A system characterised by the transfer function.

$$\frac{Y(s)}{U(s)} = \frac{2}{s^3+6s^2+11s+6}$$

Find State & Output Equation.

Sol:-

$$(s^3+6s^2+11s+6)\,Y(s) = 2U(s)$$

Take Inverse Laplace :-

$$\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 11\frac{dy}{dt} + 6y = 2u(t) \rightarrow (1)$$

$x_1 = y$,
$x_2 = dy/dt$
$x_3 = \frac{d^2y}{dt^2}$
$\dot{x}_3 = ? = \frac{d^3y}{dt^3}$ $\rightarrow (2)$

$y = [1 \;\; 0 \;\; 0]$

Sub (2) in (1).

$$\dot{x}_3 + 6x_3 + 11x_2 + 6x_1 - 2u = 0$$

$$\dot{x}_3 = -6x_3 - 11x_2 - 6x_1 + 2u$$

$$\boxed{\dot{x}_3 = -6x_1 - 11x_2 - 6x_3 + 2u}$$

[∵ we know that $\boxed{\dot{x}_1 = x_2}$ $\boxed{\dot{x}_2 = x_3}$]

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underset{A}{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underset{B}{\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}} u(t)$$

Output $y(t) = \underset{C}{[1 \;\; 0 \;\; 0]}$

check also controllability & Observability.

(5) Determine the system is Controllable and Observable.

$$A = \begin{bmatrix} -3 & 1 & 1 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}; \quad B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 2 & 1 \end{bmatrix}_{3\times 2}; \quad C = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}_{2\times 3}$$

Sol:-

$$Q_c = \begin{bmatrix} B & AB & A^2B \end{bmatrix}$$

$$AB = \begin{bmatrix} -3 & 1 & 1 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+2 & -3+0+1 \\ 0+0+2 & -1+0+1 \\ 0+0+2 & 0+0+1 \end{bmatrix}$$

$$\boxed{AB = \begin{bmatrix} 2 & -2 \\ 2 & 0 \\ 2 & 1 \end{bmatrix}}$$

$$A^2B = [A][AB] = \begin{bmatrix} -3 & 1 & 1 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ 2 & 0 \\ 2 & 1 \end{bmatrix}$$

$$\boxed{A^2B = \begin{bmatrix} -2 & 7 \\ 0 & 3 \\ 2 & 1 \end{bmatrix}}$$

$$Q_c = [B \quad AB \quad A^2B]$$

$$Q_c = \begin{bmatrix} 0 & 1 & -3 & 1 & 1 & -2 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 1 & 2 \end{bmatrix} \quad 3 \quad 1$$

* Q_c is not a Squar Matrix.
* In this case, higher order Matrix that can be obtained from Q_c is 3×3.
* 3×3 Possible Method of 20 Matrices
* Therefore $Q_c = 3 \neq 0$, the System is Completely Controllable.

Observable :-

$$Q_o = [C^T : A^T C^T : (A^T)^2 C^T]$$

$$C^T = \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \quad ; \quad A^T C^T = \begin{bmatrix} -3 & -1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & -4 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} 0 & 11 \\ 0 & -4 \\ 1 & -1 \end{bmatrix}$$

$$(A^T)^2 C^T = A^T [A^T C^T] = \begin{bmatrix} -3 & -1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & -4 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}$$

$$(A^T)^2 C^T = \begin{bmatrix} 0 & 11 \\ 0 & -4 \\ 1 & -1 \end{bmatrix}$$

$$Q_o = [C^T : A^T C^T : (A^T)^2 C^T]$$

$$Q_o = \begin{bmatrix} 0 & 1 & 0 & -4 & 11 \\ 0 & 1 & 0 & 1 & -4 \\ 1 & 0 & 1 & 0 & -1 \end{bmatrix}$$

$$|Q_o| = \begin{vmatrix} -4 & 0 & 11 \\ 1 & 0 & -4 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= -4(0+4) - 0(\ ) + 11(1)$$

$$= -4(4) + 11$$

$$= -16 + 11 = -5$$

$$= -5 \neq 0$$

Therefore the System is Completely Observable.

(6) Obtain the transfer function for the Parameters

$$A = \begin{bmatrix} -3 & 1 \\ 0 & -1 \end{bmatrix}; \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}; \quad C = [1 \quad 1]$$

& $D = 0$.

The transfer Matrix is given by,

$$\boxed{\frac{Y(s)}{U(s)} = C[SI - A]^{-1} B + D} \rightarrow \text{formula.}$$

$$[SI - A] = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} -3 & 1 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} S+3 & -1 \\ 0 & S+1 \end{bmatrix} \Rightarrow \boxed{|SI-A| = (S+3)(S+1)}$$

$$(SI-A)^{-1} = \frac{\text{Adj}\,(SI-A)^{-1}}{|(SI-A|} ;$$

$$\text{Adj}(SI-A)^{-1} = \text{Co-factor}\,(SI-A)^{T}$$

$$\text{Co-factor}\,(SI-A)^{T} = \begin{bmatrix} S+1 & +1 \\ 0 & S+3 \end{bmatrix}$$

$$|SI-A| = (S+3)(S+1)$$

$$(SI-A)^{-1} = \frac{\text{Adj}(SI-A)^{-1}}{\text{Co-fa}\cdot|SI-A|}$$

$$= \frac{1}{(S+3)(S+1)} \begin{bmatrix} S+1 & 1 \\ 0 & S+3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{S+3} & \frac{1}{(S+1)(S+3)} \\ 0 & \frac{1}{S+1} \end{bmatrix}$$

Transfer function

$$\frac{Y(s)}{u(s)} = C(SI-A)^{-1}B + D$$

$$= \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{S+3} & \frac{1}{(s+1)(s+3)} \\ 0 & \frac{1}{S+1} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{S+3} & \frac{1}{(S+1)(S+3)} + \frac{1}{S+1} \end{bmatrix}_{(1\times 2)} \begin{bmatrix} 1 \\ 1 \end{bmatrix}_{(2\times 1)}$$

$$= \left[\frac{1}{S+3} + \frac{1}{(S+1)(S+3)} + \frac{1}{S+1}\right] \rightarrow \frac{(S+1)+1+S+3}{(S+3)(S+1)}$$

$$= \begin{bmatrix} \frac{1}{s+3} & \frac{(s+3)+1}{(s+1)(s+3)} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s+3} & \frac{s+4}{(s+1)(s+3)} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{(s+3)} + \frac{s+4}{(s+1)(s+3)} \Rightarrow \frac{(s+1)(s+3)+(s+4)(s+3)}{(s+1)(s+3)(s+3)}$$

$$= \frac{2(s+1)+(s+4)}{(s+1)(s+3)}$$

$$= s^2+3s$$

$$\frac{(s+1)(s+3)+s+4}{(s+1)(s+3)}$$

$$= s^2+3s+s+3+s+4$$

$$= s^2+3s+s+3+1$$

$$= s^2+3s+2$$

$$\boxed{\frac{Y(s)}{u(s)} = \frac{2s+5}{(s+1)(s+3)}}$$

(7) Obtain the transfer function of the system following state-space equations.

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} -1 & 1 & -1 \\ 0 & -2 & 1 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

Co-factor value.

$$= \begin{bmatrix} (s+2)(s+3) & 0 & 0 \\ s+3 & (s+1)(s+3) & 0 \\ -(s+1) & (s+1) & (s+1)(s+2) \end{bmatrix}$$

$$= [\text{Co factor value}]^T = \begin{bmatrix} (s+2)(s+3) & s+3 & -(s+1) \\ 0 & (s+1)(s+3) & (s+1) \\ 0 & 0 & (s+1)(s+2) \end{bmatrix}$$

$$(SI-A)^{-1} = \frac{[\text{Co-factor value}]^T}{|SI-A|}$$

$$= \begin{bmatrix} \frac{(s+2)(s+3)}{(s+1)(s+2)(s+3)} & \frac{s+3}{(s+1)(s+2)(s+3)} & \frac{-(s+1)}{(s+1)(s+2)(s+3)} \\ 0 & \frac{(s+1)(s+3)}{(s+1)(s+2)(s+3)} & \frac{(s+1)}{(s+1)(s+2)(s+3)} \\ 0 & 0 & \frac{(s+1)(s+2)}{(s+1)(s+2)(s+3)} \end{bmatrix}$$

$$(SI-A)^{-1} = \begin{bmatrix} \frac{1}{s+1} & \frac{1}{(s+1)(s+2)} & \frac{-1}{(s+2)(s+3)} \\ 0 & \frac{1}{s+2} & \frac{1}{(s+3)(s+2)} \\ 0 & 0 & \frac{1}{s+3} \end{bmatrix}$$

Transfer Function = $C(sI - A)^{-1}B + D$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{s+1} & \frac{1}{(s+1)(s+2)} & \frac{1}{(s+2)(s+3)} \\ 0 & \frac{1}{s+2} & \frac{1}{(s+2)(s+3)} \\ 0 & 0 & \frac{1}{s+3} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{1}{s+1} & \frac{1}{s+1} & \frac{1}{s+3} \\ 0 & \frac{1}{s+2} & \frac{1}{s+2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2(s+2)}{(s+1)(s+3)} & \frac{1}{s+1} \\ \frac{1}{s+2} & \frac{1}{s+2} \end{bmatrix}$$

Therefore

$$\frac{Y_1(s)}{U_1(s)} = \frac{2(s+2)}{(s+1)(s+3)}, \quad \frac{Y_1(s)}{U_2(s)} = \frac{1}{s+1}.$$

$$\frac{Y_2(s)}{U_1(s)} = \frac{1}{s+2}, \quad \frac{Y_2(s)}{U_2(s)} = \frac{1}{s+2}.$$