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POLLACHI INSTITUTE OF ENGINEERING AND TECHNOLOGY

*(Approved by **AICTE** and Affiliated to **Anna University**)* *sky is the limit*

DEPARTMENT OF SCIENCE AND HUMANITIES
(ENGINEERING PHYSICS)

REGULATION 2021

I YEAR / I SEM

BS3151 ENGINEERING PHYSICS

PH3151 - ENGINEERING PHYSICS (Syllabus) 2021-regulation Anna University

PH3151 **ENGINEERING PHYSICS** **LPTC**

3003

OBJECTIVES:

- To make the students effectively to achieve an understanding of mechanics.
- To enable the students to gain knowledge of electromagnetic waves and its applications.
- To introduce the basics of oscillations, optics and lasers.
- Equipping the students to be successfully understand the importance of quantum physics.
- To motivate the students towards the applications of quantum mechanics.

UNIT I MECHANICS 9

Multiparticle dynamics: Center of mass (CM) – CM of continuous bodies – motion of the CM – kinetic energy of system of particles. Rotation of rigid bodies: Rotational kinematics – rotational kinetic energy and moment of inertia - theorems of M .I – moment of inertia of continuous bodies – M.I of a diatomic molecule - torque – rotational dynamics of rigid bodies – conservation of angular momentum – rotational energy state of a rigid diatomic molecule - gyroscope - torsional pendulum – double pendulum Introduction to nonlinear oscillations.

UNIT II ELECTROMAGNETIC WAVES 9

The Maxwell's equations - wave equation; Plane electromagnetic waves in vacuum, Conditions on the wave field - properties of electromagnetic waves: speed, amplitude, phase, orientation and waves in matter - polarization - Producing electromagnetic waves - Energy and momentum in EM waves: Intensity, waves from localized sources, momentum and radiation pressure - Cell-phone reception. Reflection and transmission of electromagnetic waves from a non-conducting medium-vacuum interface for normal incidence.

UNITIII OSCILLATIONS, OPTICS AND LASERS 9

Simple harmonic motion - resonance –analogy between electrical and mechanical oscillating systems - waves on a string - standing waves - traveling waves - Energy transfer of a wave - sound waves - Doppler effect. Reflection and refraction of light waves - total internal reflection - interference – Michelson interferometer –Theory of air wedge and experiment. Theory of laser - characteristics - Spontaneous and stimulated emission - Einstein's coefficients - population inversion - Nd-YAG laser, CO2 laser, semiconductor laser –Basic applications of lasers in industry.

UNIT IV BASIC QUANTUM MECHANICS 9

Photons and light waves - Electrons and matter waves –Compton effect - The Schrodinger equation (Time dependent and time independent forms) - meaning of wave function - Normalization –Free particle - particle in a infinite potential well: 1D,2D and 3D Boxes- Normalization, probabilities and the correspondence principle.

UNIT V APPLIED QUANTUM MECHANICS 9

The harmonic oscillator(qualitative)- Barrier penetration and quantum tunneling(qualitative)- Tunneling microscope - Resonant diode - Finite potential wells (qualitative)- Bloch's theorem for particles in a periodic potential –Basics of Kronig-Penney model and origin of energy bands.

TOTAL : 45 PERIODS

OUTCOMES: After completion of this course, the students should be able to

- Understand the importance of mechanics.
- Express their knowledge in electromagnetic waves.
- Demonstrate a strong foundational knowledge in oscillations, optics and lasers.
- Understand the importance of quantum physics.
- Comprehend and apply quantum mechanical principles towards the formation of energy bands.

TEXT BOOKS:

1. D.Kleppner and R.Kolenkow. An Introduction to Mechanics. McGraw Hill Education (Indian Edition), 2017.
2. E.M.Purcell and D.J.Morin, Electricity and Magnetism, Cambridge Univ.Press, 2013.
3. Arthur Beiser, Shobhit Mahajan, S. Rai Choudhury, Concepts of Modern Physics, McGraw-Hill (Indian Edition), 2017.

REFERENCES:

1. R.Wolfson. Essential University Physics. Volume 1 & 2. Pearson Education (Indian Edition), 2009.
2. Paul A. Tipler, Physic – Volume 1 & 2, CBS, (Indian Edition), 2004.
3. K.Thyagarajan and A.Ghatak. Lasers: Fundamentals and Applications, Laxmi Publications, (Indian Edition), 2019.
4. D.Halliday, R.Resnick and J.Walker. Principles of Physics, Wiley (Indian Edition), 2015.
5. N.Garcia, A.Damask and S.Schwarz. Physics for Computer Science Students. Springer- Verlag, 2012

UNIT 1

MECHANICS

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Centre of mass (CM)

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CM in a One Dimensional System

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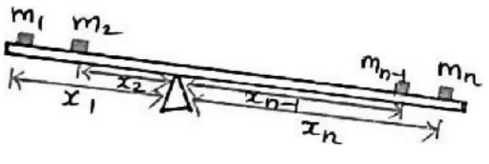

be rnez.s.s

Let $m_1, m_2, m_3 \cdots m_n$ oJ pc;;."(hcJeS- -X.. pos,rio-n o_r -:,,:. J :!.J-1-:/-'l, ·' • • • • 0 F"{hde :f'Yorn W,e. °fpo'Yllnt, pe>irit te.-r-i cle n c:., c,f c.. VVl SS t:o ot,o..i:R i <W; "'l "'-spect- b, surr C> 'tii p-O'1ot \S LOl,.\\pc.:\ "N"\omP--nt t:lf ro .

The moment of mass "i' cu, rvle.o/\ t- "!,$. l'Yl"'- Wi I'n ""{ .e..fi, pect- ••t,o the fulcrum can be written as $m_n x_n$

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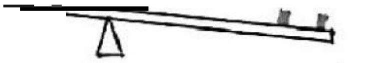

Unbalanced state

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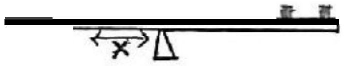

Balanced state

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Eqn ① ⇒ I"\. YY\ 'Xt .... 2.. 'f'<'\ "f-. ::. 0 (=-t L=-1

$$\sum M = \sum m_i x_i$$

l,-=-1

$$\boxed{X = \frac{\sum_{i=1}^{n} m_i x_i}{M}}$$

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CM in a Three Dimensional System:

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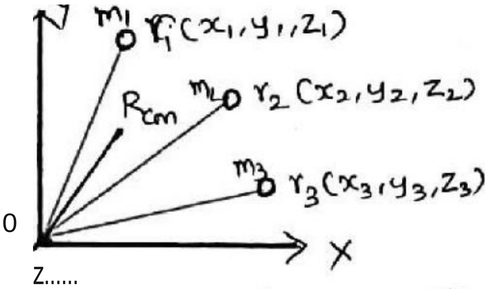

Ceohe. of Nl°' s along x-axis

$$X = \frac{\Sigma m_i x_1}{\Sigma m_i}$$

Centre of mass along Y-axis

y :

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$$Z = \frac{\Sigma m_i z_i}{\Sigma m_i}$$

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$$\vec{r}_{cm}(x,y,z) = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots + m_n\vec{r}_n}{m_1+m_2+m_3+\cdots+m_n}$$

$$\boxed{r_{cm} = \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{\sum_{i=1}^{n} m_i}}$$

where $\vec{r}_i = x_i\vec{i} + y_i\vec{j} + z_i\vec{k}$

Motion of the CM:

1he 'Motion bj'- \ .e CQV'lt"fe oJ rY\CA..S { !; no ·∂· 6LJ.,t- t.fie Jm"uz._ir-------==-==--===-===-===11

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or p"<-Hc.\ c:....:;,i \\\ "fe.C2.pect to the ca.nt'f.e of (Y) C\.e8 ,.

l,...Q_'c F → External force acting on the system of particles along x-axis

along x-axis will be

$$x_{cm} = \sum_i \frac{m_i x_i}{m_i}$$

$$x_{cm}\Sigma m_i = \Sigma m_i x_i$$

Since $\Sigma m_i = M$

$$M x_{cm} = m_1x_1 + m_2x_2 + m_3x_3 + \cdots \quad —①$$

Differenting we get

$$M\frac{dx_{cm}}{dt} = m_1\frac{dx_1}{dt} + m_2\frac{dx_2}{dt} + \cdots$$

Differentiating again we get

$$M\frac{d^2x_{cm}}{dt^2} = m_1\frac{d^2x_1}{dt^2} + m_2\frac{d^2x_2}{dt^2} + \cdots \quad —②$$

Since acceleration $a = \frac{d^2x}{dt^2}$

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Rotational kinetic Energy:-

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Here, the velocity 'v' varies with radial distance from axis xx'

v_1, :i,.1 ,J-.... · · · f be linear velocities "(Y\ , M1 · · ·

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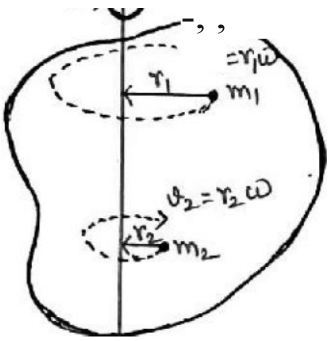

K, E of p Cl.'Y J-;c.le_ '''f.
o.t Mc:t gg YY"l1 $= \frac{1}{2} m_1 v_1^2$

k• E. o} p:i--"f:n e,\e.S $= \frac{1}{2} m_2 v_2^2$
of \'V1el-Gg Yv1 L

$$\left.\begin{array}{l}\text{Total K.E of all}\\ \text{particles}\end{array}\right\} = \frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2+\cdots \quad ①$$

,yeloJioV\. b<2.h-ot.2..ex\ L 'Y

v e-Qo ci +:, o.r1 4 o-18\ v.la. v Q.. toci

\ fjVe'''\. 6

O\::. 'Y W

$\therefore$ Eqn ① becomes

$$\text{Total K.E} = \frac{1}{2}m_1r_1^2\omega^2+\frac{1}{2}m_2r_2^2\omega^2+\cdots+\frac{1}{2}m_nr_n^2\omega^2$$

'Tot<.l K- o l M;r_;jc,:i'- -@

The body by 1-fior<\,Q,\l'\r of ir1ertt{c.. o:F c,.,bO-l-e, l1'.L i-- o..ti:i S ½ ft'\J

$I = \sum m_i r_i^2$

:. '1ri. (3) t•E = --k_l cD J :z-1

The above represents particles

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Theorem of Moment of Inertia

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Parallel axis theorem:-

Theorem:-

It states that the M.I with '''(e pec\- to o..''' cuds (s €9\.\.0J 'co fho su.CV\. of rnO'O\IL.'''- t or i \''U 'ln o'-

w, \ i" e.spe-ct h) o-- po.."(O...Hd ci.."tis p,-- i"8 ffi-ro c'''' Y'.d P,oJ.uc..c- of f'Yll\.$S (;'.I"\d .Squ.o..:re of \ij .e. pe "f Pe.n d.1(>.{\. Q.".Y d. i.& h:u'\CQ... bQ,twa.e.,n pa-"(Q.t\e..t '''-:s: JZS

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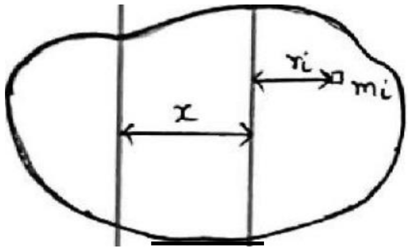
Proof :-



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xx' axis

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d ' = '<""""'Y\.

M.I of the entire body with respect to xx' o...f..l is

$$I_{xx'} = \Sigma dI_{xx'} = \Sigma m_i r_i^2 \quad ①$$

\\'y

M.I of the body ·0-bout AA'

$$I_{AA'} = \Sigma dI_{AA'} = \Sigma m_i(r_i+x)^2$$

$$I_{AA'} = \Sigma m_i(r_i^2+2r_ix+x^2)$$

$$I_{AA'} = \Sigma m_i r_i^2 + \Sigma 2m_i r_i x + \Sigma m_i x^2 \quad ②$$

Substituting eqn ① in eqn ② we get

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$$③ \Rightarrow \boxed{I_{AA'} = I_{xx'} + Mx^2}$$

This eqn represents the eqn for parallel axis theorem.

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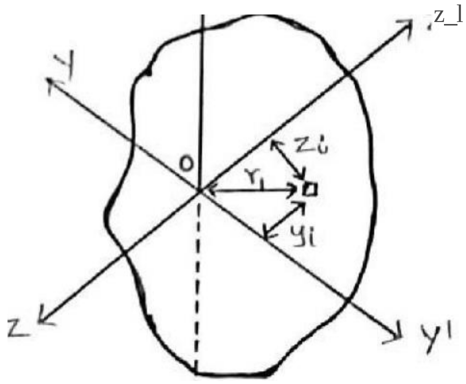

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$$I_{xx'} = \sum m_i r_i^2 \quad \text{--①)}$$

From the figure, we can write

$$r_i^2 = y_i^2 + z_i^2 \quad \text{—②}$$

, u.b-st-;+-v..tind n@ \M eq"'O) we. o..ve.

$$I_{xx'} = \sum m_i [y_i^2 + z_i^2]$$

$$I_{xx'} = \sum m_i y_i^2 + \sum m_i z_i^2 \quad \text{—③}$$

We know that

M.I of ,p\ ne. o-.bov..t 'YV 1 o... 1.s \n 'L

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$$I_{zz'} = \sum m_i z_i^2$$

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M.I of a Diatomic Molecule:

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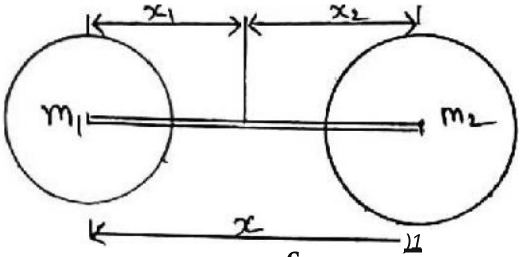

0 -> C-M of (\u 5jsre.rv_

'X.t x.2..-> 'J)is+o.V\CQ. oJ b..oo a.tt><n$ 1 1 1 {"fOM, 'P°l n1, O .

From fig, we have

$$x = x_1 + x_2 \quad \text{—①}$$

Since the system is balanced "-oi l'f} 'Y".e-S pe cJ: to the CM, we have

$$m_1 x_1 = m_2 x_2 \quad \text{—②}$$

From eqn ①, $x_2 = x - x_1$ —③

Substituting eqn ③ in eqn② we have

$$m_1 x_1 = m_2 (x - x_1)$$

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$$m_1 x_1 + m_2 x_1 = m_2 x$$

$$x_1 (m_1 + m_2) = m_2 x$$

$$x_1 = \frac{m_2 x}{m_1 + m_2} \quad \text{—④}$$

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m, t:X..-:c:2..) m2:::c.1-.

$$m_1 x - m_1 x_2 = m_2 x_2$$

$$m_1 x_2 + m_2 x_2 = m_1 x$$

$$x_2 [m_1 + m_2] = m_1 x$$

$$x_2 = \frac{m_1 x}{m_1 + m_2} \quad —⑥$$

M.I of a diatomic molecule about an axis passing through the CM is given by

$$I = m_1 x_1^2 + m_2 x_2^2 \quad —⑦$$

Substituting eqn ④ & ⑥ in eqn⑦ we get

$$I = m_1 \left[\frac{m_2 x}{m_1 + m_2}\right]^2 + m_2 \left[\frac{m_1 x}{m_1 + m_2}\right]^2$$

$$I = \frac{x^2}{(m_1 + m_2)^2} \left[m_1 m_2^2 + m_2 m_1^2\right]$$

$$I = \frac{x^2 (m_1 m_2) . [m_1 + m_2]}{(m_1 + m_2)^2}$$

$$\boxed{I.::..- \frac{m_1 m_2}{m_1 + m_2} x^2}$$

Since $\mu = \frac{m_1 m_2}{m_1 + m_2}$ called the

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Rotational Energy state of a Rigid, Diatomic molecule :-

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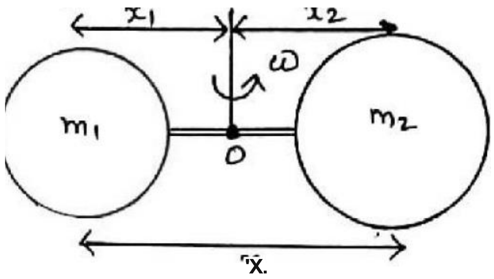

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w L :r::, —②

Substituting eqn② in eqn①

$$KE = \frac{1}{2} I \frac{L^2}{I^2}$$

$$KE = \frac{1}{2} \frac{L^2}{I}$$

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$$L = \sqrt{J(J+1)}\,\hbar \qquad \text{-CD}$$

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£L.6s.l-i tt,....n .cl V) ® eqn ⑤

We. –e._1 'L

$$K_E = \frac{(J_c:r\{;r\text{-}tD\ \text{-}\backslash iJ}{2\mu x^2}$$

$$(ie)\ \boxed{E_J = \frac{J(J+1)\hbar^2}{2\mu x^2}} \qquad \text{—⑦}$$

E=.qn @ Yep-Ye.Sen t:S. Yo m.H on a.J i +orn1c::.. N1ol.a...cu..\Q... 9 h..V.-vi tvu2.,chAP\cl adLO .

. o.f s

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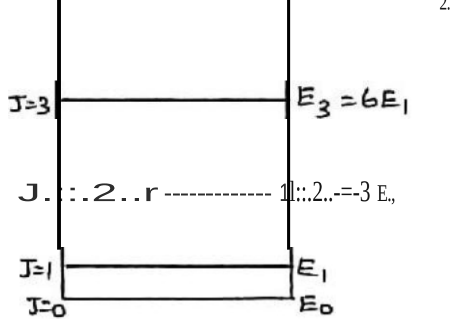
$$\boxed{E_J = \frac{J(J+1)}{2}E_1}$$

J";,..J r------...JI½ =- :rc..:r+o '=, 2.



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Torsional Pendulum

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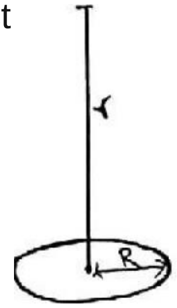

C → Couple per unit twist

But applied Couple = $I \frac{d^2\theta}{dt}$ —②

Where $\frac{d^2\theta}{dt^2}$ → Angular Momentum

I → Moment of inertia

"l..n eq w \;'o--<h>--"'-"

/.\pp\leo\ Co"f'°-= 1-o'<i" L...

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t}J.:..'1, :c.

gioc..e, a_cc "'fo...h"on \5

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∴ Time period of the oscillation $\Big\}\ T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$

$$T = 2\pi\sqrt{\frac{\theta}{c\theta/I}}$$

$$T = 2\pi\sqrt{\frac{I}{c}} \quad \text{—④}$$

Determination of Torsional Rigidity:-

- * The disc is rotated through a small angle and set it free
- * The time taken for 20 complete oscillation is noted. From this, the period of 1 oscillation is found.
- * The diameter and mass of the disc are measured.

W.e.k "'ou-:> c.'

-r-=- '.l-1\J -®

qu.o-.."<"'fina on bo s1d.es

$$T^2 = 4\pi^2 \cdot \frac{I}{c} \quad \text{—⑥}$$

Su.bs-\-itu..\-f""a Couple per unit twist

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'Kn,o:-<YO-\i"\'a- ' o..,bo\J'2. eq r"l

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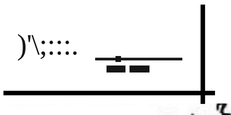

Where $I = \frac{MR^2}{2}$

M-? MA S of dis(.. d\iv....s. oJ \e disc_.

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Double Pendulum:-

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be.ho-Ve o.. o- Sim pe_ncl.u\ .

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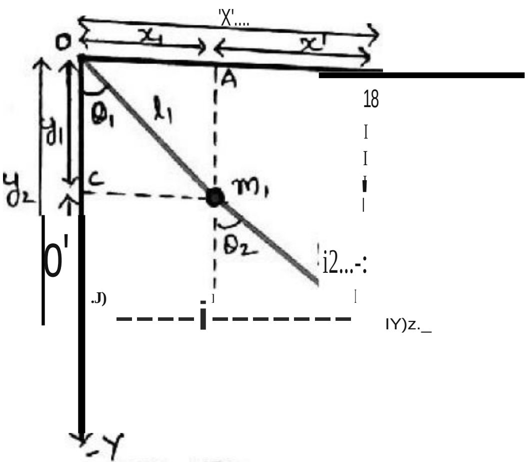
it be.h. Ve.S 0...S o.. c..ho..oh"c. s:,s1'eM.,



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$$\frac{dx_1}{dt} = l_1 \cos\theta_1 \dot{\theta}_1$$

(ie) $v_{x_1} = l_1 \cos\theta_1 \dot{\theta}_1$

$\vartheta_{y_1} = l_1 \sin\theta_1 \dot{\theta}_1$

$\vartheta_{x_2} = l_1 \cos\theta_1 \dot{\theta}_1 + l_2 \cos\theta_2 \dot{\theta}_2$

$\vartheta_{y_2} = l_1 \sin\theta_1 \dot{\theta}_1 + l_2 \sin\theta_2 \dot{\theta}_2$

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Kinetic Energy:

K.E of the system is

$$T = \sum_{i=1}^{2} \frac{1}{2} m_i \left[\vartheta_{x_i}^2 + \vartheta_{y_i}^2\right]$$

$$\cancel{T} = \frac{1}{2}\sum$$

$$T = \frac{1}{2} m_1\left[\vartheta_{x_1}^2 + \vartheta_{y_1}^2\right] + \frac{1}{2} m_2\left[\vartheta_{x_2}^2 + \vartheta_{y_2}^2\right]$$

Substituting the expressions for respective velocities, we get

$$T = \frac{1}{2} m_1\left[l_1^2 \cos^2\theta_1 \dot{\theta}_1^2 + l_1^2 \sin^2\theta_1 \dot{\theta}_1^2\right] + \frac{1}{2} m_2\left[(l_1\cos\theta_1\dot{\theta}_1 + l_2\cos\theta_2\dot{\theta}_2)^2 + (l_1\sin\theta_1\dot{\theta}_1 + l_2\sin\theta_2\dot{\theta}_2)^2\right]$$

Potential Energy:

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Moment of inertia of a circular Disc

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h,...v'Ch °'-of o.. ci-f cu..\0-Y. cts, e..w c \ l,j

'ooto--f"\ a \.5. at various positions

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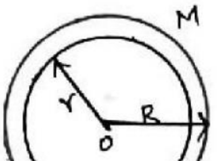
Mass (dm) = Surface density × Cir. of the ring × Thickness of the ring

$$dm = \sigma \times 2\pi r \times dr$$

$$dm = \sigma 2\pi r\, dr \quad —②$$

We know that the surface mass density



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$$dm = \frac{2M}{R^2} r\, dr \quad —④$$

gw,s\-i@Ii 8 e,n,. Q91"' eqv, <D

c\:l: -=- ,. d "'1' • '1rz_ R.'2..

$$dI = \frac{2M}{R^2} r^3 dr \quad —⑤$$

Integrating eqn ⑤ within the limits 0 to R, we will get total MI of disc.

$$(ie)\ \int dI = \int_0^R \frac{2M}{R^2} r^3 dr$$

$$I = \frac{2M}{R^2} \left[\frac{r^4}{4}\right]_0^R$$

$$I = \frac{2M}{R^2} \cdot \frac{R^4}{4}$$

$$\boxed{I = \frac{MR^2}{2}} \quad —⑥$$

Eqn ⑥ gives MI of circular disc when the rotating axis is passing through the CM.

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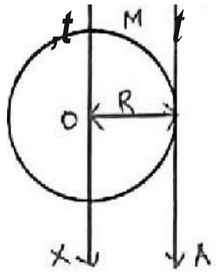

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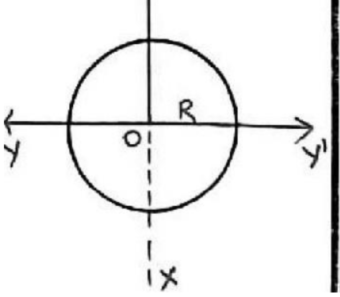

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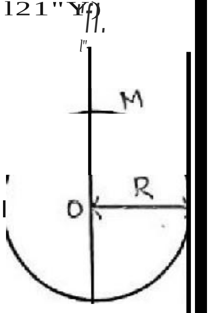

$$I_{AA'} = \frac{1}{4}MR^2 + MR^2$$

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UNIT 2

ELECTROMAGNETIC THEORY

Maxwell's Equations

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Derivations

cf Mo....""-wel\l:. E-<1v..o..nons

Maxwell's first equation from electric Gauss law

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$$\oint_S \vec{E}\cdot ds = \frac{Q}{\varepsilon_0}$$

$$\oint \varepsilon_0 \vec{E}\cdot ds = Q \quad — ①$$

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$$Q = \oint \rho\, dV \quad — ④$$

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$$\oint_S \vec{D}\cdot ds = \oint_V \rho\cdot dV \quad — ⑤$$

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Differential Form:-

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$$\cancel{\oint}\quad \boxed{\vec{\nabla}\cdot\vec{D} = \rho} \quad — ⑧$$

Eqn ⑧ represents the Maxwell's 1st equation in differential form.

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$$\phi = 0 \quad -①$$

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Differential form:-

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$$\boxed{\nabla \cdot \vec{B} = 0} \quad -⑬$$

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Maxwell's third equation from Faraday's law

:cording to Faraday's law

$$\varepsilon mf = -\frac{d\phi}{dt} \quad -⑭$$

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$$\oint_l \vec{E}\cdot d\vec{l} \quad " \_J(B-<As) \quad d$$

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$$\oint_l \vec{E}\cdot d\vec{l} = -\oint_s \frac{dB}{dt}\cdot ds \quad -⑰$$

Eqn ⑰ represents the Maxwell's 3rd eqn in integral form.

Differential form:

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From Ampere's circuit law,

$$\oint_l \vec{H}\cdot d\vec{l} = I \qquad —(21)$$

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the current and current density given by

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Substituting

$$\oint_l \vec{H}\cdot d\vec{l} = \oint_s \vec{J}\cdot d\vec{s}$$

1'3 £\--o S -re.WI

$$\oint_l \vec{H}\cdot d\vec{l} = \oint \nabla\times\vec{H}\cdot ds \qquad —(24)$$

Comparing eqn ㉓, & ㉔ we have

$$\oint \vec{\nabla}\times\vec{H}\cdot ds = \oint \vec{J}\cdot d\vec{s} \qquad —(25)$$

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Here $\vec{\nabla}\cdot\vec{J} = -\dfrac{\partial\rho}{\partial t}$

$$\therefore -\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\vec{J}_d = 0$$

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Plane Electromagnetic waves in free space

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$$\vec{\nabla}\cdot\vec{E} = 0 \quad — ①$$

$$\vec{\nabla}\cdot\vec{B} = 0 \quad — ②$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad — ③$$

$$\vec{\nabla}\times\vec{B} = \varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t} \quad — ④$$

The wave equation in terms of electric field in free space

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$$\vec{\nabla}\times[\vec{\nabla}\times\vec{E}] = \varepsilon_0\mu_0\frac{\partial^2 E}{\partial t^2} \quad — ⑥$$

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u 1---;, Fr,,
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Compa."fina eqns CQ ,® Loe :f'"

$$\cancel{\vec{\nabla}\times[\vec{\nabla}\times\vec{E}]}$$

$$\vec{\nabla}[\vec{\nabla}\cdot\vec{E}] - \vec{\nabla}^2 E = -\varepsilon_0\mu_0\frac{\partial^2 E}{\partial t^2}$$

From eqn ① $\vec{\nabla}\cdot\vec{E} = 0$

$$\vec{\nabla}(0) - \vec{\nabla}^2 E = -\varepsilon_0\mu_0\frac{\partial^2 E}{\partial t^2}$$

$\therefore$

$$\boxed{\vec{\nabla}^2 E - \varepsilon_0\mu_0\frac{\partial^2 E}{\partial t^2} = 0} \quad — ⑧$$

Eqn ⑧ represents the wave eqn in terms of electric field in free space.

Wave equation in terms of magnetic field in free space

Taking curl on bothsides of eqn ④ we get

$$\vec{\nabla}\times[\vec{\nabla}\times\vec{B}] = \vec{\nabla}\times\left[\varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}\right]$$

$$\vec{\nabla}\times[\vec{\nabla}\times\vec{B}] = \varepsilon_0\mu_0\frac{\partial(\vec{\nabla}\times\vec{E})}{\partial t} \quad — ⑨$$

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Wave equation interms of magnetic field in dielectric Medium.

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Using vector identity, we can write

$$\vec\nabla\times[\vec\nabla\times\vec B] = \vec\nabla[\vec\nabla\cdot\vec B] - \nabla^2 B \quad \text{— ⑪}$$

From ⑩ & ⑪

$$\vec\nabla[\vec\nabla\cdot\vec B] - \nabla^2 B = -\varepsilon\mu\frac{\partial^2 B}{\partial t^2}$$

From eqn ② $\vec\nabla\cdot\vec B = 0$

$$\therefore \vec\nabla(0) - \nabla^2 B = -\varepsilon\mu\frac{\partial^2 B}{\partial t^2}$$

$$-\nabla^2 B = -\varepsilon\mu\frac{\partial^2 B}{\partial t^2}$$

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Energy content:

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Tot. Energy Content in EM waves } = Energy content due to elect field + Energy Content due to mag field. — ①

Energy content due to electric field.

I ....J k \,: c-1'\.Q..., tf'd !:i, tia oL.u, h,

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— ②

Suhsl--J\''lA..liVl5 Solc..d{c of wave

(A..9,.l)OVJ Jfl Slrtll. _p:>YM_.

(ie) $\vec E = \vec E_0 \sin(kr-\omega t)$ in eqn ② we get

$$U_E = \frac{1}{2}\varepsilon_0\vec E_0^2\sin^2(kx-\omega t)$$

Let us take the time avg. of energy density to find the energy content.

$$\text{(ie) } u_E = \frac{1}{T}\int_0^T \frac{1}{2}\varepsilon_0\vec E_0^2\sin^2(kr-\omega t)\,dt$$

$$u_E = \frac{1}{2}\varepsilon_0 E_0^2\frac{1}{T}\int_0^T \sin^2(kr-\omega t)\,dt$$

Here $\frac{1}{T}\int_0^T \sin^2(kr-\omega t)\,dt = \frac{1}{2}$

$$\therefore u_E = \frac{1}{2}\varepsilon_0 E_0^2\cdot\frac{1}{2}$$

$$\boxed{\text{Energy } u_E = \frac{1}{4}\varepsilon_0 E_0^2} \quad \text{— ③}$$

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$$\therefore U_B = \frac{1}{2\mu_0}\vec B_0^2\sin^2(kr-\omega t)$$

Let us take the time average of energy density to find energy content

$$u_B = \frac{1}{T}\int_0^T \frac{1}{2\mu_0}\frac{\vec E_0^2}{c^2}\sin^2(kr-\omega t)\,dt \qquad \left[\because \vec B_0 = \frac{\vec E_0}{c}\right]$$

$$u_B = \frac{1}{2\mu_0 c^2}\vec E_0^2\,\frac{1}{T}\int_0^T \sin^2(kr-\omega t)\,dt$$

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°''''eA. Ee:-.. _;-AoC-'L we can write

the above equation as

$$\text{Energy } u_B = \frac{1}{2} \varepsilon_0 \vec{E}_0^2 \frac{1}{2}.$$

$$\boxed{u_B = \frac{1}{4} \varepsilon_0 \vec{E}_0^2} \quad — ⑤$$

Eqn ® "(e--p"'e.se.11\t.S 1w.. er'\Q.:(

Co"'lı--eŕ"ı in cA--...-o f'() r,o,K c. luWJ

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Total energy ,,",,'_' - _" re>'\'r due to both fields:-

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Intensity of Electromagnetic waves:

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Derivation:-

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$\theta = 90°$ tt> eO\..d, o!"J It:i.e,y

$$\therefore \vec{S} = |\vec{E}||\vec{H}|\,\hat{n} \quad -@$$

$$\text{Since } |B| = \mu_0 |H| \quad \text{O'r} \quad f\backslash 1l = .!j1 \; J_{,,} 1/4$$

$$① \Rightarrow \vec{S} = |\vec{E}| \left|\frac{\vec{B}}{\mu_0}\right| \hat{n} \quad — ②$$

Solutions of wave eqn in sine form are given by

$$\vec{E} = \vec{E}_0 \sin(\vec{K}\cdot\vec{r} - \omega t) \quad — ③$$

$$\vec{B} = \vec{B}_0 \sin(\vec{K}\cdot\vec{r} - \omega t) \quad — ④$$

Substituting eqn ③ & ④ in eqn ② we get

$$\vec{S} = \vec{E}_0 \sin(\vec{K}\vec{r} - \omega t) \cdot \frac{\vec{B}_0}{\mu_0} \sin(\vec{K}\cdot\vec{r} - \omega t)\,\hat{n}$$

$$\vec{S} = \frac{\vec{E}_0}{\mu_0} \cdot \frac{\vec{E}_0}{c} \hat{n} \sin^2(\vec{K}\cdot\vec{r} - \omega t) \qquad \left[\because \vec{B}_0 = \frac{\vec{E}_0}{c}\right]$$

$$(\text{or}) \quad \vec{S} = \frac{E_0^2}{\mu_0 c} \hat{n} \sin^2[\vec{K}\cdot\vec{r} - \omega t] \quad — ⑤$$

∴ The time average of poynting vector shall be written as

$$\vec{S}_{\text{Time ave}} = \frac{1}{T} \int_0^T \frac{\vec{E}_0^2}{\mu_0 c} \hat{n} \sin^2(\vec{K}\cdot\vec{r} - \omega t)\,dt$$

$$\vec{S}_{\text{Time Ave}} = \frac{E_0^2}{\mu_0 c} \hat{n} \cdot \frac{1}{T} \int_0^T \sin^2(\vec{K}\cdot\vec{r} - \omega t)\,dt$$

$$\text{Since } \frac{1}{T} \int_0^T \sin^2(\vec{K}\cdot\vec{r} - \omega t)\,dt = \frac{1}{2}$$

$$\vec{S}_{\text{Time ave}} = \frac{E_0^2}{\mu_0 c} \hat{n} \, \frac{1}{2}$$

$$\vec{S}_{\text{Time ave}} = \frac{1}{2} \frac{E_0^2}{\mu_0 c} \hat{n} \quad — (L)$$

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Eqn ⑧ represents the intensity of electromagnetic waves.

Momentum of Electromagnetic wave:

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(i) Momentum in terms of energy:-

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$$\vec{p} = \frac{uk}{c}$$

Magnitude of momentum

$$\boxed{p = \frac{u}{c}} \quad —⑦$$

Eqn ⑦ represents the momentum of electromagnetic waves in terms of energy 'u'.

(ii) Momentum in terms of Poynting Vector

We know the poynting vector

$$\vec{S} = (\vec{E} \times \vec{H}) = uc\hat{k}$$

$$(or) \quad u\hat{k} = \frac{\vec{S}}{c} \quad —⑧$$

Substituting eqn ⑧ in ⑥ we get

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p =- t C 'L-

Si n c.e_ C".L... =- J___ fuf"o

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Radiation Pressure of EM wave:-

Definition:-

ˈl-J\, ef'l ela-ch-o (\Q,.-kc l.J)..Ve Ab-n \.:;e. su_-<f-c:-.Ul., 1°kV"l Q... j&(CJt wl \I "'-f>pea.y clwt.. +o '1k c:J-io.ncp- of t'nol'YlQ.nh.uv,. ""Th..c.. CJJ'C"\co.v. \- bf pY.e-SSu_Ye. exe-d-e.J.. pev- \J,.N\1 l:: lX"leA. on hv\.'< 'V'Cll. to p"iC. 1!:. CA\,e.9 "ro.cti°'nol'\. f"f2 StA."f e.•

Derivation:-

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S= u..c. -®

Acco y-cHn o -to po n ti"' 0, el"()./ e,ta.ctrro m°ij -h'c.. '2.nQ.. YfJa P"- t;C () ()oy ('()o-\. CO S <..L'Y"f,._tt pe.'Y UM 1t a yea._ °'- 11d o f'l\.l2_ v, er ve "- bO

$$S = \frac{u}{At} \quad —③$$

A A."<eo.., t, -') b" M.Q.. LOMpO..'YI(\d .eq ® ®

t.A-C. = .At.

c:. =- L/Ac

$$\frac{1}{c} = At \quad —④$$

Substituting eqn ④ in eqn ① we get

$$p = uAt \quad —⑤$$

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F - v.-At / t

$$F = uA \quad —⑦$$

wkt, the radiation pressure Prad exerted on the surface is given by

$$P_{rad} = \frac{F}{A} \quad —⑧$$

Su.b sri+u..H n O the eqn ⑦ in ⑧ we get

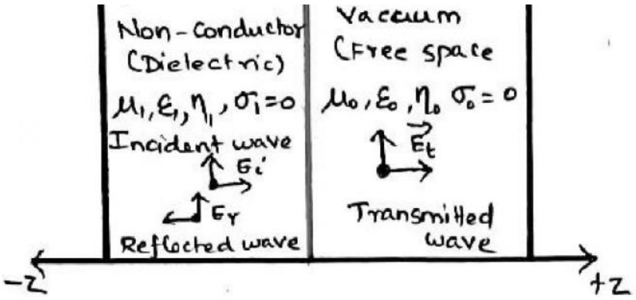
P-...oJ u... A·

ijr-r u.. –®

Prom eql""l ® we.. co-.v"l so..1..1 cu: J - "frA_oio..\iOt"-. tp"<e.SSU."Ce... oJ e,T'r\ v.:>C...Y C2. 'r 1-> e>avJL\ - I \::.o ,e,.f'UtYQ-.,j u U e.,ke.,t'YOl'Y">°'fj l"\0.-ti"'C.. t.,lO. VQ.

Reflection and Transmission of EM waves from a Non-conducting Medium to Vacuum:-

Consider an EM wave travel j N""l o.. 11ori- COn d.u-c..n..,..a m V C.U.l.lJ'il' p:it- 0 f f e, j 1\C,,\ cl.D_V\ wo..v'2... r9,, '"f.e..c. d, n to "hO..r-Af_ 1 v..m AJ-- i't"l e'f 1r tt o...n cl AA'\-0 \ pru. t- ..,; +--r-o..n rn, 'r\-ed 'i\ next medium.


Non-conductor (Dielectric)
$\mu_1, \varepsilon_1, \eta_1, \sigma_1 = 0$
Incident wave
E_i
E_r
Reflected wave
Vacuum (Free space)
$\mu_0, \varepsilon_0, \eta_0, \sigma_0 = 0$
E_t
Transmitted wave
-z
+z

$\varepsilon_1, \varepsilon_0 \rightarrow$ Permittivity of non-cond. medium & vacuum.

J>-, , }J₀i 'Pe."f n'\.Q.C'-b\ ,l;. o:f- non_
Coni:\-. VVUt.di u.. ht. Vc..c.uu.tV

, GY 6-li2c..• f.e\d,_ -JectoYi; o.f
1ncick{\l:- 'i:,l. 'Yej-Lu:}:i.i.c.\ u.»..\Jes

Ht, th- M°t}·..fe\c\ \.le.c...+c,..,,- o]
i!'\Cl d&,l'"\1--- YJ. "(e.. c.red. WctV

W e.. c.o..f'I LA.'.) YI t"2..

E.L +- Ev ::. Et —①

Ht+- \-\y::: t. --ID

L RR-p-r-.Q. ev"\-:S l<1ciclii..'i\t" u..:,o...v12.
"'f" ..!:) R.e..p1""-Se.l'\'t-S '<e..fsi.c\-24 wo..V
'u f""<e V\.,\:S t'YOX\9--n', ed tuo..v'<2..,

J2. f)Smissio11 = d

Leeri, iot"f"\n \c. \rf'lpe..d0-'°'c·e
1 + (I on - c.on,.d l"n..lu::llwv'I·
n ·,nt"fi..,.;.tc. iMpe.do.nc.'2.
' Lo
of Vc:. L.1-UJ'/\ ·

W\<.c (1_ -:. J;,.. 9 \,\ :: –®
H l'\.

rto -:- E =}> \.\ =!... –®
l-\ t\o

Uglnfl " !.l® '2q'1 (J;) C.0...1"\.. k

WYi-ffer-i a.._s E.t - Ey - Et::
Ti rt. rt»

-Ve. g8(V"l '1od,'c..o. e$ tA.+- 1t\.Q., Yglec.m:\
WtWe. l,yo,.Ve\.9 (V"l 1ita. oppa \ t°'2. d..\'l'eqio,,
t0 ,ttid- l.:r:) {Kt. \(\CAJ..c_ t- wo..e•

j_(_Ec-E,-) - Et;/'lo
Y\._l

EL,- 6"(-::. fl, 6' -(8
- !:;

Adding eqn ① & eqn ⑤ we get

$$E_i + E_r + E_i - E_r = E_t + \frac{\eta_1}{\eta_0} E_t$$

$$2E_i = \left[1 + \frac{\eta_1}{\eta_0}\right] E_t$$

$$E_i = \frac{1}{2}\left[\frac{\eta_0 + \eta_1}{\eta_0}\right] E_t$$

$$E_t = \frac{2\eta_0}{\eta_0 + \eta_1} E_i \quad \text{—⑥}$$

$$\boxed{\frac{E_t}{E_i} = \frac{2\eta_0}{\eta_0 + \eta_1}} \quad \text{—⑦}$$

T"'fe\.0Sm1ssio<" c.oc..f:f,de...,..\- fs
"fo..ho \e ,ore"' '1':\ of \\;,e
.\-"fo..,......SN"'l',\\ d. wo..',{<2 c:t.1;-) -n,
\n.\-en s\1a of iV1 t_\O,ll.\"\'r 1.J.$.,..\/e t.:ri)

' :Ct —⑧
Lt.J2.) , - :t L,
I.;:. c:::L

\Nkt \,

I(\h2n i o:f }.Yo.1'19',.,·1\\e-c\1. :t :: -6:
V..:0-\le_ J t 2-'lo

I " h, V1<SI111 of I" Cl'al,." \- L I = i t ---{[;
v.:x>- e J 2 'l,

Substituting e. V'S ® bt @
'L

T = Et /2-
sf/z.tt

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$$T = \frac{\eta_1}{\eta_0}\left[\frac{E_t}{E_i}\right]^2 \quad — ⑪$$

Substituting eqn ⑦ in ⑪ we get

$$T = \frac{\eta_1}{\eta_0}\left[\frac{2\eta_0}{\eta_0+\eta_1}\right]^2$$

$$\boxed{·4'\backslash..(), \; - \; ti.. \; (\_t'\backslash 01- <)V} \quad —®$$

£q<\ @ Ye..p"'fe.Se""·\-S. +"fo.n mi io

en ef-fi i (2_1"\ t-i,

Reflection co-effic_'e. lR)

g,u.be.h'N h" a .e_qv; in el\V"\ CD

we. ax2-c;

,t-- Ey -== 2'\o E-('\ot-'\

$$E_r = \frac{2\eta_0}{\eta_0+\eta_1}E_i - E_i$$

$$E_r = \left[\frac{2\eta_0}{\eta_0+\eta_1} - 1\right]E_i$$

$$E_r = \left[\frac{2\eta_0-\eta_0-\eta_1}{\eta_0+\eta_1}\right]E_i$$

$$E_r = \left[\frac{\eta_0-\eta_1}{\eta_0+\eta_1}\right]E_i$$

$$\boxed{\frac{E_r}{E_i} = \frac{\eta_0-\eta_1}{\eta_0+\eta_1}} \quad \_,S$$

71'.e. Ye..f{o.e-n or, Lo <2--,ffi ci e,V\ \-- i S

ycJio of i nh2n si½ of 1\::ie_·

Y ft:a c.re d.. w ve L:!-1)

1n+er1 i¾ of ,rid V\t wo-v (jy

$$(ie)\; R = \frac{I_r}{I_i} \quad — ⑭$$

Intensity of Reflected wave $\Big\}\; I_R = \frac{E_r^2}{2\eta_1} \quad — ⑮$

Substituting eqn ⑩ & ⑮ in eqn ⑭

$$R = \frac{E_r^2/\cancel{2\eta_1}}{E_i^2/\cancel{2\eta_1}}$$

$$R = \left[\frac{E_r}{E_i}\right]^2 \quad — ⑯$$

Substituting eqn ⑬ in ⑯

$$\boxed{R = \frac{(\eta_0-\eta_1)^2}{(\eta_0+\eta_1)^2}} \quad --®$$

f::-9ri @ p-Ye.' ,er.·\-5 fla..c.J' -

Coeffi <:J e V\.-)--·

The sum of T+R :-

i+R @ -+@

$$\therefore\; T+R - \quad 4\, Y\backslash 1> \; \eta, \; o'T\; l'\backslash J + \frac{(\eta_0-\eta_1)^2}{(\eta_0+\eta_1)^2}$$

'2.. n..

4(\ o --1- 'i + C'\-1 2- r'\.,<\,

_Y\o -\- f"h)._

$$\frac{\eta_0^2+\eta_1^2+2\eta_0\eta_1}{(\eta_0+\eta_1)^2}$$

T+R -:: + r'\,J L

o :-:\ \)\.

.". [-r R::- ']

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ctnd .f-,ya.nSMiSSlo"" Coe.ff,e."'f-

et\"'-"" 10 oru. ·

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UNIT 3

OSCILLATIONS, OPTICS, LASER

OPTICS

Total Internal Reflection

When a light wave is Completely reflected while it travels

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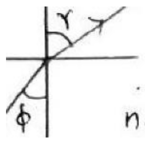
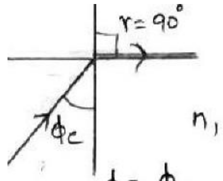
e. \\:<!d, 1;: 06:,.\ ·,,.,,h2Y'no-.\ 'YQ.f-loc\-io,

'\'h...



$\phi < \phi_c$

Fig 1



$\phi = \phi_c$

Fig 2

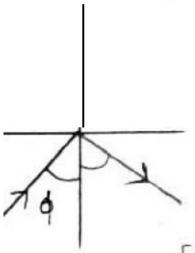


Fig 3

$\phi > \phi_c$

Let the light wave ~~passes~~ travels denser medium to rarer medium

, h.a.v1 1tt.a. o.n ()le.. of rric.iJ&f"lCR::-

) rS ui£S tt, "' cy,t{c...c..\ O.1 _

l_lSl.) < c. r -wie. 'f O..'.::) i.3

refracted in to rarer medium. Fig(1)

Case(ii)

\,•.H" f"t'\ = c_ ,11,e 'Y°=J plA.<:.S.'e S

o.\of\ tv'ic;,cliuW\ of 92pa."< hon o..S

SbolJJYl If"\ BB 0--) ' £o ·W!.c.. \-- (Anale_

of "'fe{ 1e1.c:ho1"1 15 9J ,'ThiS o. le..

(: \S coJ\eol Q \"'1'hca.l 0._vt;fe

Case(iii)

When $\phi > \phi_c$, the ray is totally reflected into the denser medium

·1t tf o.,; <;hown lM --:r 8(2,J

.f='YOC"Y\ Sfl<? \\ 1 S (CALO

'()! (1\ c_ :: fl2-<sl () '1° 0

$$\sin\phi_c = \frac{n_2}{n_1}$$

Critical angle $\phi_c = \sin^{-1}\left[\frac{n_2}{n_1}\right]$

Conditions for Total Internal Reflection:

Condition 1:

Light should travel from denser medium to rarer medium.

(ie) $n_1 > n_2$.

(one;\; \7on i

)½ cv,cfD- of rAciJ-2.f'IU. at

\(\re,fc-C'2_ shov.\d \:,a_ aYe c...rev

CY°itic_0,,.\ 0\1"'\fl\Q. •

) '> (.....

Interference:

When two light waves Superimpose, then the resultant

°'-N'f)'l e O"r i"' tQV1S·1f1j lr. P...

're fon of £Uf><?'Y pb$ di Or'l d;:ffl'QwI

·ti,C\n <Ampli de. of 1nd1vicR.\..\L'\.\

"'--». VQS ·

Corv ; hor- ft,Y Co0 h·v1d'l\le i<"lte-<reYQ.n :-

tl 1/,\e,;v c.. O.'Yf' i0 p <;e aV"\d hC\ve_

p t',,<e, ;ff eYe c.es of- O, 2'.ii; ½-l•l• •

(_li) 'PC\ti. ;,rf1?'<"n02.--= (\t',, n f".::.O,l,2-

Jk c.{s lla. po-th ot,ff eYeV)c.a

t).Q.·h,<>02..(")1k '1\-ifb W°'-VeS . 1 S; u.D..l

+ ·(\h?8'-yo-..l 'f<')u.JhF Dr

IAX'Vs e. t..e."' 1fi C.>-) , C...0 S!--')-"\Ac:.

3 -+RQ V) , n h \Je

f<""l\-e"(fe '{ e. V\(.Q O CCU Y ,

C.0,=hho ,... %,Y ol.es 1-"""Y'A c.h v.e \ n\-e-r_r-€11(.Q,

d ') vl 0- v .es C\.--i e. 1('l o'-..l\- oJ ?h o..S.e

0\v.d hcxve phO.SQ d. •ffe-{e,,, of

1T , 3fl i i;-11 - · - • -

/\

(l°1) p c1,-rf eie m :: 2--

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lfuA.b ' cl { P°'l J.;ffe-<Q.<"l to..

t)Q.tw €.Q."1 1v-fO '--l.l0..Y: e£ 1.£ 1.,1..Q\

h, c,::,\ \vir-e.r-a.\ IYH...J..b"t,LQ cf A.

2-

'''YU. c\-ive 1<'l re'i fe""e."-CD.. occu..Y.S.

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'TT&

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'$,uch tho-.\.. o pl 1l-lfni'.) r 1 '<5 '7'1",

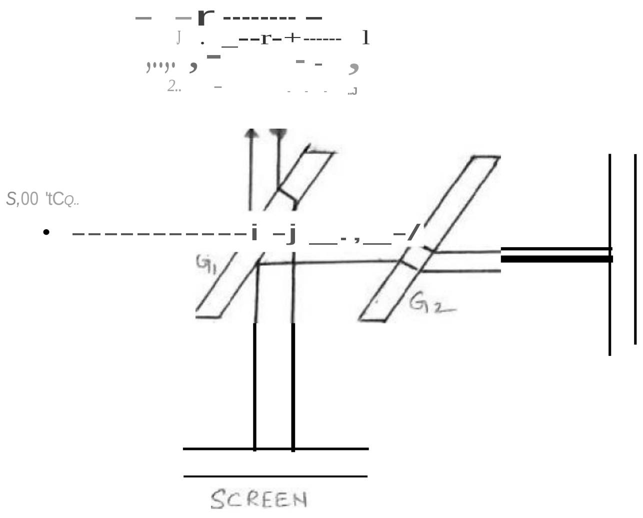
·the G5h.. f'Yo\""(1 {),._ c,u-Y ,5,

Q. uci.\l '.:j "< 0fi..0 cJ--c:,d c:,.. n al- 1 ,YO l'l ,. mi H

!:>'j. a,. "3 (1 w0-J.J o,.; vi ,;i ol"

o.r<p> I; ,tv..d...Q.. to. lea -pl o. Ul. •

S,00 'tCQ..



Michelson Interferometer:-

'Froc£u c..·1('c3 i ri re'f_fe"'fe"'c.o. pa- .(!.-n-i

6d sp\1\\-i..-i5 V\ 9Jff-r bea.N\. 1n-h:i 'f\.,J\J -

f""O> .,_.,q fu,, "<'2 W" bl ni"' ';l

1t'e,M o..frey --9'c-.ve ,}-r•ro..V(2.\\e.di

o\ ffe v-v,t opnDAI po-.i'.-hs -

Construction:-

.l -t- eons 1 £. ts <::>f- o... 'Nl 0vc-..\d..Q...

T'()1'YY0Y fV1, . O..V')d "" Ji '°'d 'rYl'l"tYo'f

ot?i c;-,.-v.e. \--iif'\a f'ol,s J.

* Tl.\..l? tj o..g£ pl N""e1 6,1 c...6e.avn

gp\i\te'i) o..nd qi_ (SP..

crf"-S p {o..\--e) aye_ p\vxCJz.q f0"1a.\\e.,

iD eAc.-h otfu..,y- b'2--h...vea. ,

Working:-

-* f'l\0 0 0 C..\.-,'YOl'Y1 0..h C.. \ l e;._\l,..,-\- f "'rOIY") 5 C'\J.:HR..

J'C'-"S (?\) if)e_ t)QCH'(O) 5 §1l;j: y S?lkS_$

O're G\,, u

* Sin c.e Bi, is pcu.\\ o' \\'-J pol, s·kq

,,g,ort1e ,po:<b of- l"k Gff.r ts

'Xe,--\e.c\-'2.ci o..nd 5o!Y\e.. po..•--.+ of

t; t + 't'Yo..vi. rni Mee\

pmctucino tw-o !=><2'Y p<2n d,ic..u\cw

be£,\,N)_S of ua..l I0 T'eV\.Sl .

1k. kc\-e)..j.'-11- \- 'fn oV e ·u,.dwd

M , °' '(\c\ +-rr a Yt.Sm i.\l e. fd- 'l°'r)o/e

tnwo..'<c\s M2. -ro a..Po..ss

lo..\-e 2, o·

M, c,.nd l'll,._ '<e\l c.\ tka_ 6

6<:,-ck 1 u.:0-Y,I. beam splitter G.

inc ing reflo< I ., from M, is

l t \, f \:p 0. vv)

– 7'rn S m I l-1 d

11-11 "(n.., ·tr.:: r<eov'. o Y)d tRe.

"Y refle ed from M₂ is

lected gain by beA.rY'\

p\1 4-\-ey to ifu CJY eQ.n •

&, CXQ.. Q 60 \ beomg 0Y15irol

· 'o0\'Y"1 tf\Q g(Xrnt2.. po\ 01 on thQ..

,..;ou.'rG2., 0-,L(_ Co h.O!'< el;'\t O. Yld

+Rev e [6va.. \ "\-e"i Cx v0l\ \; eo-c..lA ⁰

A0. ·1,--J·e·{\ e --rQ"" CQ. 1'>°'--\\e Yn of-

0."' 'K °' ""d b:,'C t -f,¥+. Yi "" S o'Y

I I nrye o.Ye ob.Se."(V e.d on

Vl Q w I"b· SC. "f e..Q.(\ 0..·\- S,

Theory :

F-Yin cs D-JJJ - ·)'(V) e b flv.;

J 'Y J ec:1 f TY\ 1rrroY

v1 o..,.,d \V\2- 'c.h rs e9u..ivo..lo.v'\t
1

..t: '-1>f·t YQf(i?Cl::Qd f'YD"" V--PP""(

0V'0 11..,,_,_,c,y ,tu-'Yfo..CQ Oj (k o..,·y FIN\ I

form t;,r b lu...Q.Q.l"J "fY'l\"'Y-rD'Y IV) 1 O\nq l-'li

theQ v, 'ih.A 0-I ! rn f/'l ..c of rn i '"f'Y o"t' M2...

ince the two interfering

beams light were split

from same initial beam,

they hally in prase.

or a given separation of

'd' tween C;_ '"fY1, ...- ,-o'!'S l'Yl 1 o.n c{

the Vf ,,, C "f{\1"<-Y Y t"' 2 1 r e l r;;,o.J\;;

diff '"(Cl\" 1.9 (C, r'

2d = nλ

Ld\-lv i O ·,n ll) (1� I integer,

J, fe'fo'('ILQ _/ » n the two rays

'f'('IOV n '1,

Q:l..,v1 t>e ti o "Y1 Pd bd

The two waves will interface Constructively or dostructively as per the following conditions of path difference Δ.

1 1

i) Δ = 0 (no path difference, no interference pattern)

c1'1) 6:. n>-- C (ons\--'Yu.C..hve 1-tire-Y-
2- –frrenc.Q.;- BYI 5 p,if"I

8 \ 'i) :kJ-Q+D)-. ('J)e.51'Y li-l C.+-1v-e l <l \-ei eY.en
2. _ '1)c?(i 'Yl () 5-C.$.

Types of fringes :-

The fringes -|c')' rned. l:, '-1 'J

0. iy- ·lrn "rn b2. ,.... ,'"f O-A\Q,"f Cu ved

(:yJ 5 'Y\:).8J. t '

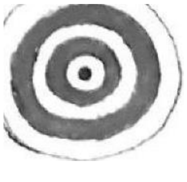
Circular Fringes:-

[\1£1

K\,e (\ t<'\, "' J.. o' PC\

h) -o. Yi ·tf"li' C i "I' L\.\ \.

f Y1 C c; -o. t)(' 'I\J



vecl F

W\"leV"\ M1 ondt;-1\ Ol"fe_ tfldtr\QQ +0 .Qo.d, oiheY 1 /1)Q f \l'Y) enc::..QdEC\ l wed ,;hcfecl 7Tu:i-- cu"<"e f\--1na-Q..,.3 Ll,;lV\ b(:'l c¾seY\Jed '

M, r-1 2,

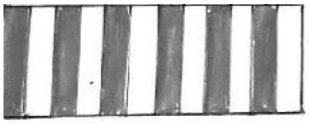
In--r:irr-

Straight Fringes:

1-J t'½ M, O,.<l o\ M{ in te'Yi:ec.t-; Sf-"'cu t \iV"& f'Y,n G1Ye_ o6-ku.x1 d ---

----- --::::::::._-:--_ M,

-



,,...---y--...-.,--....,,,-....,,,-----v"------.f"--.......,,,,.--)

De..+ <10.\--i Df

from

ve-ty e. Po--"f()._t7o of J befwC2.Q.V) M, o..vtcl 'i-4 \ p-- diff eYe.V\ CQ_ v) 3'1"en 6. '.:1

2.J = (\ " -{D

When M1 is moved a distance 'x._l €-0\.C pi() QVQ..,S -\-0 '1t;e__ pog,f,O() p'YQV1'ou..s.,l:, occu.pfeg bJ CH') O clAo.ttn f cri ('l'}e, li2 t m 6.:2. YlLl.Mbey Dj f--ri"n sc;fno 0.. 3'vQn point t<t\ \1 mt:JJQ:J

P°'\t-, dJff eYeY1c.Q. h:Y 'f<)D\Jll"o \"fu.,. M\'Y"YOY JS, <zr"e.v) '2..C c\+x.) (Yv1-t-1"1) 1'- -@ '). c\ +1 ,x_ = m)\ +n /l -@

Substituting eqn ① in eqn(3)

$$n\lambda + 2x = m\lambda + n\lambda$$

$$2x = m\lambda$$

$$\lambda = \frac{2x}{m}$$

Applications:

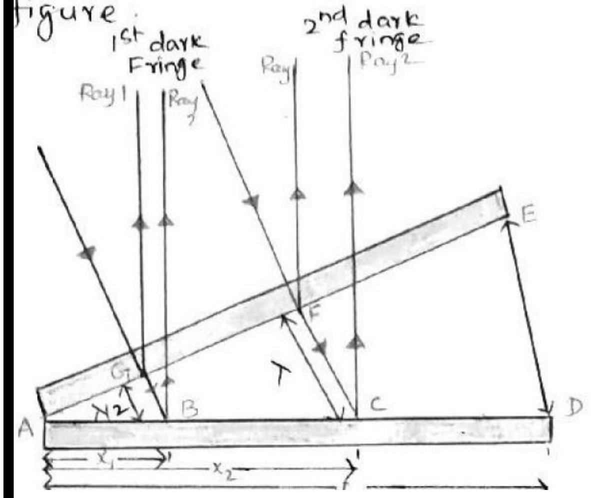
:t, It- d-o.\-ed-ed. e>Cl \--eV1CJL c5 l.<-XAV .e.S o. Y1d C..On frY Ni t2d £-po--CQ - bi MQ d Sn:,'(fion /. r+ is u.£.ed in o pf-i co..\ lo -YeYl CQ_ +omo7J3o..ph0- :fr .Tc ,s. u.sed. 1(") ffoe'(ophc..s.

Theory of Air Wedge & Experiment:

A 1"Rfr1 f fM o..v,n 'Z..e..o0 -ck- 0-.t 01'\Q e.nd. Ou'\ c,\ po0 tl' vd.6 '1n <:.'Yeo..9..lYi.c) 'c., Glt- tf\e. 0 .-W-iey e0tc::\ IS CC\.\\<29 CA \JVQ..d_,

Th.,£- o.i -Y v-J<2d.3-°- c.o..v\ 6e we_ +0 t11'1 fu_ _.CP--Ve.(il. \ or (fu. \ (\ d.Q. \- l\f\::;. lV\ove. \mpo"f\-0-1'\ r½ c.:\ eo- be. 'iJ$J2,.O. -t-0 u 'Ye. Slisz. Qt Ve'Yd J-,Ma..\\ o c. -

The arrangement for observing interference of light in a wedge shaped air film is shown in figure.



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\ \;e o)"'f \M, n ., rVI'l 'i m· \eel

'thYVu h OJ."l t1M, Jfui. -\;fo-riS-

N"I·, -" .e d, t)Q.o..N'I 0 8-9';,l"l 'Yef Co- cet

ot bo +t-ory\ of rk oJ "/ J,'\fYJ •

we2ddt2_ o.n \I) ve.o

ji,m,s11, 0-n J, 1-\," ,t;t,.l)O "(.e..j'(Ochocj

'Y n ...,Jes e_ Con J..,d;wJd or

MIY'v\ cHIJ4J p-roc:hl tJO o.ilevnt\f.e

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'Y\ (") Q w Id.\·\, ?-> ::- X)-.. X \
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$$\boxed{3 = \frac{\lambda L}{2t}}$$

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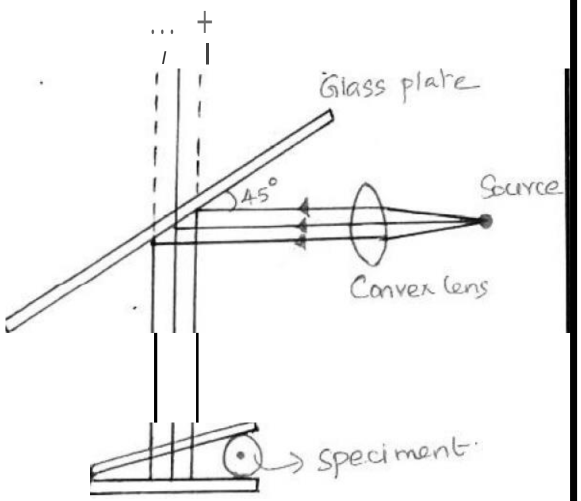

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* V4Yel '" \"\-, e. \ i5) o..\\S ()C5'{rrc-J\ d
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o\,ue.·\-o ·1n\-eYfeY.Ql'I.CQ. 6e4uJ02.V'I \\;:\sl,
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wkt

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$$\boxed{t = \frac{1'}{I}}$$

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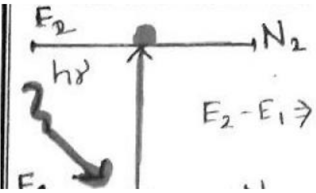

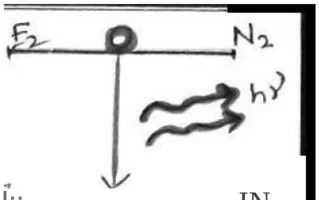

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le.J N-Sf-t N t No...6 ©,

sub eqn ①②&③ in ④ we have

$$B_{21}N_2 + B_{21}N_2Q = B_{12}N_1Q$$

BI'). l'-\, - R.l--\ N'1-Q = Al-I Nl-

(9>,1-- N, - 8>11 N1-) Q A0-, N'L

11 = " " ' = " " " : ; ; l i ;

i A'>-, t-J 2-

Bn.. N1 - 1?>2., N'l-

N l,.\.m-6-YY ole, om w,MO Y

by $B_{21} N_2$

$$Q = \frac{A_{21} N_2 / B_{21} N_2}{\frac{B_{12} N_1}{B_{21} N_2} - \frac{B_{21} N_2}{B_{21} N_2}}$$

$$Q = \frac{A_{21}}{B_{21}} \frac{1}{\left(\frac{B_{12}}{B_{21}}\right)\frac{N_1}{N_2} - 1}$$

From Boltzman's equation.

$$\frac{N_1}{N_2} = e^{h\nu/k_BT}$$

$$\boxed{\therefore Q = \frac{A_{21}}{B_{21}} \frac{1}{\left(\frac{B_{12}}{B_{21}}\right) e^{h\nu/k_BT} - 1}} \rightarrow ⑤$$

W· .-\i,o..f- OJ..t..ci'rcU "1{o 'P t,lc,

f2R \Oh

Q "" Is11 h>1 __!_____- -'7,J,, l7"'i

c.} e.ri->'/ ,- I

CD .,....'fG..tri °1\M. h'o"' ®

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B.;i.l

Az\ _ f;1'h./, (QY) _ -_ /"i @

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Conclus1 Q 1v... A,,,ti Oh (j)

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Nd-YAG LASER

LASE:

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vfit\ y , v. rri A. \i..,1.. m, m1,1 vY\ Cn C,vYn P t"

CHARECTERISTICS:

Type: Four level solid state laser

Active medium: Nd-YAG Rod

Purnf°111 mQ..TY1 : 0 P\:,1 c£L.t l>--r0p1 nt:;
C..,;,c on \._Ar0p)

ROJ. on tor : ,tL>--\ \j poGt \---eJ evn o\
rCV°' hc ..-l½ paLl d W!
t-,._\ c\ - "-/ A- Rod

Pw / ou..t.PtAt: 70 yJ

N o.-•h.µ.{'e... pv.-1 1---'2J (ii J u,1-n \',H)\IY
\,,\Jet.A, e_ -foT (Y""

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cl:V

'The, Y1.C2ocL. m; vm "Ct,tom +wY'f\

}-jol - '-J p,-Gt Yo J. 0 p ti CllL ll j f' 1Ak0 p,o\

6 ><.evi on J cY'.:J r ro}1 fL.t....d,.., 16.kn P

:PIA.Kl na_J \1-,e_ t'v7.-,-. 11L1 on tr,rom

m-etu _s.t-cd:iI e. S\-c1.. te- tu 5rt)IA.ri ots+c :ie.

!v-r 6.QtLl'Yl 6b wle..v,51-h f-o

P'Yl"l .-{d £, \---\-e.o/ ,

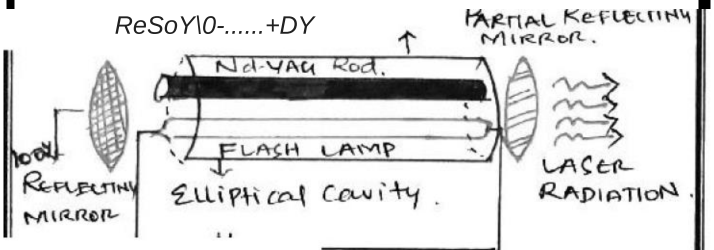
*4J t--11.:n I.Jc..not--\
h.e. Nd-'/A&r L, '_)J\A.\ C.u..r f)1 -tne

frn-m of c11-t.£,\ de Y ...

cVte- w13 --\:j po u rwcl SO M b:i be

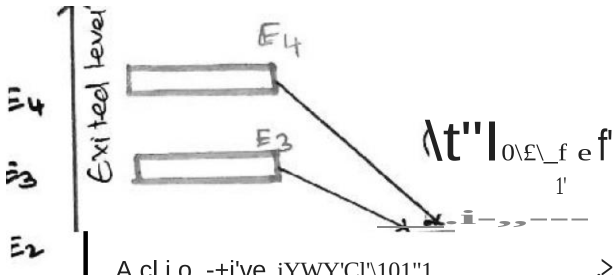
o \J ti ca..\\ f' lo....-t ct, o 1.,,;_d,, d .

c\. n.o\e.Y1UL\ rod OVY\d. °'-
.f \ \,\ \D...XYfi C 9u...m p1 !'oU-k ce)
ke.pt v a\ \.e,,\ **D.N"O** \:'\cu.,e ot.
lfhS.ic\e ·O\ Yef-\ee,,tor- CO-..N'it-j
t2-.r1cU; of 'h"\e, Co..-v, Co\LQx-ec;f
'o j I oo '/. '.Pcuv.\ioJL PDW
y'Y'\\)Y"oYS wkH:...°n?t.u\:S CU ev-
ReSoY\0-......+DY



+,1,1

ENERGY LEVEL DIAGRAM



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Ci..) Di.A. to o...b.co -pH on o+- U t
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0-IY\. 0. 'f?o l>"'m No\ t- r-\.eocLm1\A.VY>

atoms are exited from

Eo _::::; E..; OvV\..ot lf . \-e..v..12.-ls
<J0 :l pon44.½eoW S'/on
o..-toW'l !, { rom m-ov:e.s fa
£ '.:L -t-'vie ry,Q. . ,t-c._s -\- le+ c·t..-te
a.,\: t,,oYV'\ D v-..\ QV) \ 1'1Y--<½1t'DYJ
j-.t, o._ Q..Nl 1 Q>.(..e '

I l) B t:1>bi mIA,\.J e oQ ['J.!Y,V\ c; io Y)
1'rovY\ 1'-\-1on E:;2..- E:., ih.t.ic.Jra
Le.eccl to L C\..c...t1'oh 1 Q-1
I-A Yn C\o boo X) le½e.."' t
_µ \-\eJ ..
G v) Thcie... Pho-ton..c 'f" \.e.d.eo\.
b Dl-tK OVn.cl -fay -h-. 612-t vo.e..-t..k) 1\0€,.
m'\ Yror S \J Y\o\.Q.Jy3 o A i i coJ-1 or)
r...Vn U2 hA ; n te.viu l0..._-'e...,...-l '-' t:
Y'n1 t-\e 1YJ YO V--j \,, fC.d)'il \ \ 'j
:\10\A Yl..f) d Yh \ rYD Y'1'

Advantage.

H, 3 -e_,yJ :J o l p
R.ef-€--t,.\-\oY\ rl\ ..-.i-€... AAV h.A
'PoPu.J.. ti oh ·, nV-e,yi, o n Cl Gh.A e.Ne
vU1½i

Applications

l) .e& otA °'- Y-O-IY\.5e ,Rnc e.,v,. 'fn
fY\i l\ 1 , \-J
\.J cf +oY- C.LL-+t, n.ai dY1\ \iner
e-1 n.j t?\IV'-.ct WY f8-u h..cvv"'5
l
VL l nJv.AMJV.•.
Uf.ecf Y Ceda..v-a.c.t \A-'Yj0Y'j,
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1n.... 1--..r\.ed; Cov1 ,P,.e...1d ,
U .e.d l Y\ lo':{ \., cLJ...<)
Comm unication.

CARBON DIOXIDE (CO2) LASER

:-:T1 0,.._ --r-t ou-Y \.Q,,v..e... \ Mo\.e..c,l-'-Jo. Y
,, Gicw LM e Y 0 p..eY2=L-te '(\ ' +o.,,v-
T r Ir
..-'0.-p-C(..y{2J Ye..j l Oh .

CHARECTERISTICS

Type : Four level molecular gas laser

Active Medium : t,./\ i x.e.-... 0-b C.oi , N 1 , 1-\e CU) -

Pumping Method : Jn elM-hL c-.1-01Y1 -I,0 C\.tom col\·,non.

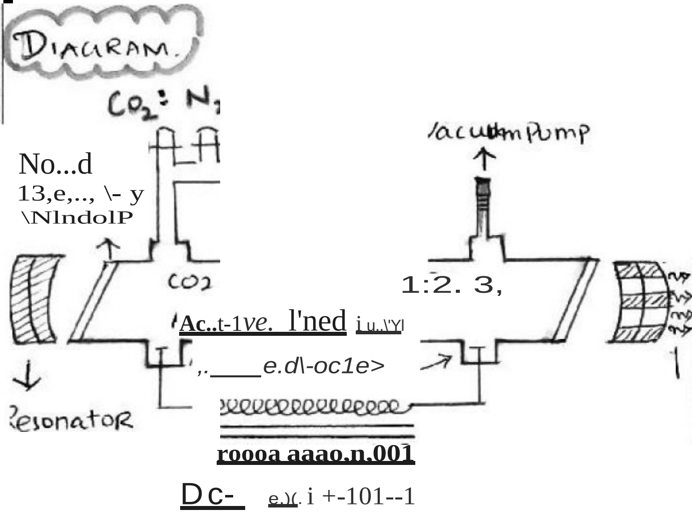
Re.ioncdo, ! -F(..L.\\_j Po Jhed P?vr t'l c...(L:J rt,Ll...che o Coh (..£.t v-e_ rn ·,r v-- o .-- r ,

Pw /OP : 40 w/m - 60kW/m

No..._-\LA.Yl2.. i Pu.._\'2-1' &")') lonnniDIA.t;.

WC\..V..e.. {-e..n th : vt. b prv1 10 n--. •

DIAGRAM.



·{1:'t CoY\ 1(-\--.s A di c\'Ja.y5e_ tLL of Dlo.Yrl--e...-\-.e."r),- S" c.. vY1 o..h e,\ Le.n...5 t; (YI -

\1) Co?, N'l.- 1 H.e.. f\ o-t 1Y\.c.,1cte. ¾e + L,L\? e_ 1 n. --t1r).e ya.Ji o I •• '2 1 YR-l IO , Yupe c...\-1 v-e½:1 •

(I i ,) P if c. en.cur e..1 fYDJ L.LC..O J b .'.::f D c..- ..e..)(\ + -\-7 on.

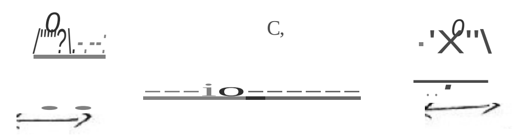
0 V) N ctc..l - Irv i Yldow > ON'€... p\o..c.e.J 'f Yl e,,Ll V\-Pd cx...t. P.,Ye.L--C'9''''t.e..) .e... fO tv"le... Y onA..'toY o...x. \S •

(y) 0,-,.. t o,f -tne. ..\- l-'--v.,e eov.e,y..Q.d w i POJYho..l -f:u-l\'j o \A, h-e0 Cor. c..o....Ve,.. \,\\ 'r'rOY-S' . W i \\ °'-C-\- q,_ ..::t.. 'Y onA+o'r.

VIBRATIONAL MODES

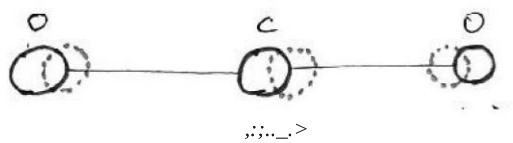
C l?;;. "11 o\..e u ..,..A.e._ 1 Ll V'l..e. c..,y h""°' o \Q e-v--1..e. C-onf,1 ins oj CeV\-\YCJ. o---tom w,-\vl t.wo 0"). 'j'j.enC'c+orn0 on e,lh-ev t:;ic\e,;. 1\:: CoY> ist- oJ 3·1h cle f-0¥1o\d--1A--t V1nYCL.ho tr,oc\eS·

- ,,,.Y i.C..- -...,, t re_C



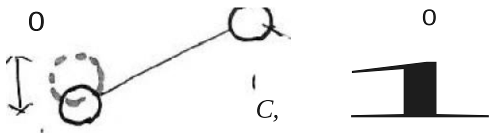
J'Y1. ll/\.1\g, m_oJe. CoY bon o...tDl'Y' 11 5 tcd::.\o ncvv·'::j W h0-re. +vi-e. -two O ?lj °'-.-torn V, bYO-te., o.JonJ tv.-e. C-\..X 1 . Lrea ,\-ye +...ly.

(ii) Assymetric Stretching mode



.1'n -\"i-\A > mode CA.\\ -tv\e +rw·ei2..... o.....---\om5 (0Sc.,i\\o...,te.. ll.\on1:J ¼e.o.. 1 .. 'o\A.lc.. me. V, \?yo.,_non J_\\ C-\-" on Oi (O<.JY' bo n ·1.S oppos, t-e i-o m·C\.t o:f me ox vi.

(iii) B d,yt mode



it\.t lY\oc-\e C\. \\ i1ne +h Y-e..R..cz.+om tl .eKjo V1\oYv,..-\-i t\l\ f..e'Y peo/\ot ,c uJ.evy --1 0 tY\e \?0Y1.o\ 0-x, g O\lv\d tvie re.- 0) mt>Ve.m0V\. t oJ 1Y1em oppo 1k ko e.o...e...v- otne-v ·

Co). L Q..y.. i-u.c\ b<_j °'--

Dc exitation

(i) The exited electrons collided

\AJ ൃ1 N ; YD ccf"''' vv1 o\.e C-LI 01 \.e_s b 't

w c.\,\ N 1- caot e.,;1; \:ae to

Me--\r--1,_5.'tC<-lol.e s "L- e..

$e^{*} + N_2 \longrightarrow N_2^{*}$

e 'tp t 1'-1? Q '''i ½ hru_ 1>>C\J

5 .\-r te_ CO 'J.- YY) o\.ec..1.,v -e_ 'o j

1 tcq\rc Col\, ion -

$N_2^{*} + CO_2 \longrightarrow CO_2^{*}$

iii) due to this Population inversion is achieved in the

E4' l.e,v.e. l

j·¼) (?::, <;·h n, v.,\ te c\ (; 0 v

ı t--f'o\ cv--\e; - mc...ke_ A..

'\YcvY' 1\-'\on tr°M E-4 to E3 f

1i, yl? \.A-)(,, v0 \.,v c.,,l,--. P\ (D\,u_,y6AAJt-

WOv'T' \. loy-m Ofru>\

l1 6 }Am ..u '1\eo\ 're.,.q>-u-<hV-V.j.

\h-th.,e_ IR on _

1:_\I) r. I 'flbV\ E ?- -'7 G.I

f ?:> --3> G I Vl-'1-<--ff-€AAs otL,L.IL-to

S{ ti L Co\ \ i (Ns\A •

(vi) ''CJ L,V.N\ to

cl.Lrvh tvu- h..D_a.ut m Co'l-

to- Co w\,e +o _1'Yl? I .e Ea

Jn\--&vv---n;,ryvic: ee,)l,J l\'.M

MERITS

* Construction is Simple

* O/p is Continious

1\1 C''' izfJ1c 0V\J_

.,(ve y._j H , :;- 0 / f Po w ex •

-" 0/ f Pw ((AN', he. ·,Y1 th (Jew>J'

b'j r] r (,v' QCc11 flj' 11/\€ tuig tvi c hu

t LA-h--e. •

DEMERITS:

* Carbon monoxide will contaminate the Oxygen.

Depends on Operating Temp.

f" c..e i t » ' ''' l f< r CY\ it is

, y,\/1<;i b\e.., Au--1 W

t,()C po v-,ye_ s IJYV"€_

s.e,N- JV) Ct- vr, ct.3-c__•

g

tt- w·,ck u c. 1 n 'V\cc-\

? YO tR..,H; \---j w,e) J' ,.C(\,..\\ \ihj ,.Cu.l-\1 j

o\d-e v-\ Y\.J .

..,_. v1..! .Qe.1 fo..- OpeM CV-y Co r,'lv'-1 co

U f Q J , ·YI Ye,-V00 .e_ S12,,vlC, 1-'-_3 .

* l t Ll V'-V d I ''' -t'h e_

Treatement of iver q Lunch

r:z=f f 1-\ E1

tO·bt'

-¥ 1 t,, (A»J t 1'y –

'''J/1:... <I ,bI-'''' 1- I': 1=:i. IC Yv fv.,v-;iV'''J b \oo d

::i 1' --:-________e, \.D,c ws e:, f-v : d

_J,..;,. w?- ,1., , Eot=========d

Solid - state .Sem fc.ovwlu t-r.>''' r:D tc:id e)_o,Je_

:rt o... >'Y1o.st c..o l'V\f' o.t t -fin--h'l 0-+ ocU. l cu. e...,.-

* It also called injection laser.

* It is broadly classified into two types.

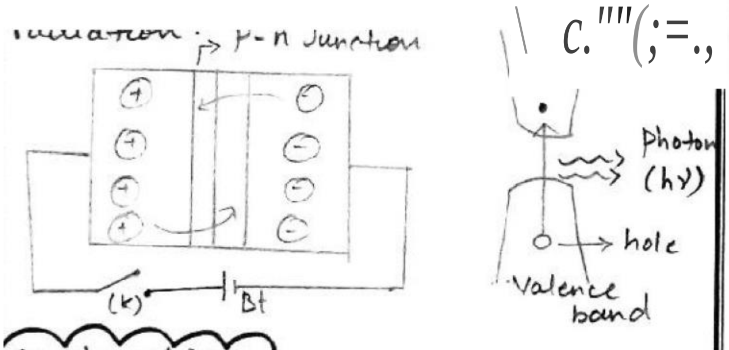
QI.) tt.0 n-U:1Jt..t.vt C Heh>'YO

Homo - Junction Laser :

It is c.fo.t .-f-¢. bevi co.t e.J. pN" -;J'I.LJ'lC d.i<9 cl e_ .

Principle :-

when the pN Junction diode is forwarded biased, due to recombination t,t"' e.lec rv&IA- o.w:l kole.J ½\ttl --rodl h .Q..vyu tte.d.. r n. oLl.,.- .d.- bawl Se,&l\U C.Cl'\.dt.tM"O"t k.k.ow-«.- cu 'YC2.CDh'l bf



Construction :-

* p-type and n-type semiconductor combined +0 OA... J>-n j'LU'll'

* two e. .l ec h1>oleA. W"'-'1.eel-'2A. ei--11\.. =p and bo ttewt .:Hcl..e...

* prnvo."'J.. b..to..A-l .V Si v'2VL rrt>Ujk O\el'_h-vJ .

* Tu -e. €M..d. -ba..c.u 0t _p. ,J_ c..u,..cib:&\lL GU..t. we-1.l b -1 :.l,,.._ 1.-Jtu:ck ac ti cu. an optical resonator.

Working :-

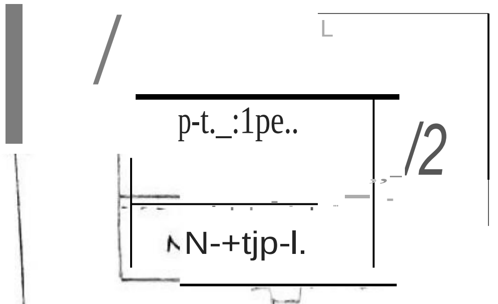
e.. c...b-c.ui.-l J'o'Y WOry-i:{ bicu.QJ As &ht, w-vi.. ,.L,,\.. 113<1 1.-e..vel -elec k..lrf.ei a--<e. f"(je fn.to J &v\...

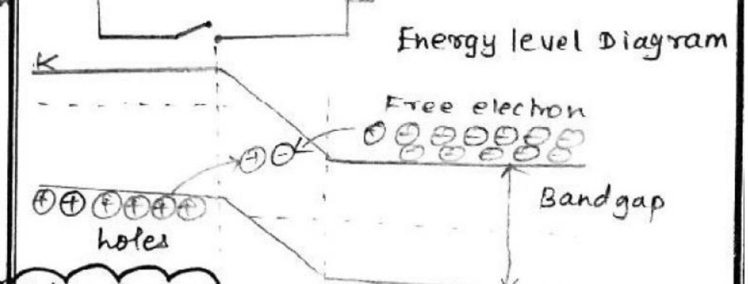
* 7vie. J.a,,.... e. ri..o- ofr e..,lec e,_g Ye.00wtbi"'e. u.,'i'-+tt_ €Qck O'tt.tl_y.

* Due to --tt.u.A f +tHu 06 h -9 en.J tt-"c!

* This llh'e..cl J,tf e \, side of d..toel e .

* The /vt1rre -+trte biasing voltage more emission of photons are p6)-f.s.i ble.. .





Energy level Diagram

Advantages :-

→ gyy.aJ..l a..w:l lo c
→ HI-H3 h e.1 pu· t-
→ O 'Ytl ted.: tiJi<ttA. less_pOlJ>e, compared to other klcu,
→ Output is continuous / pulsed

Disadvantage:

→ output has l ari- e.. d..i. ver-<'1U'IU

→ poor coherence Cl.V\A. swbil.i.

Application:

→ used in b <2 "lf or,li t C...0yt"I h'lU--W Ol •

→ used in printer & CD p l e.,u.

------ , ----------- , ---------- 110u J:-t

Hetero-junction ...

In this, p-n junction 1)0'"Yrned.. a.i'Y<I'ct

by more than 0-\11. n-type p-type material.

Principle:

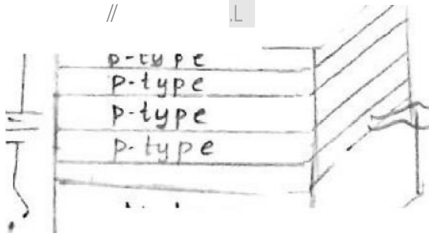
when e f"- JUY1c+te-vt c:Licde _.r,. .r;F.- r,.. c,.r,-w___"1" b 1, cf e0L

forwarded b..uu t\.,e. e./e l)}a.ve.) '9o A -

'Ye.co b1he i-o d..uu f>hc -

Construction: _L"'_--

// .L



......._N.,..•_,i.-.!j p, J, /

ct- Tu p- pe.. 4vu a.nJ__ two

n... p 4-v-u ton,,,bl e. 6o ..fc-rm.

p-Y\ J L - nett 01/l .

... the p layer in the

J c..llOVl Q.. b a.A 0..W. active region

... ius ... laced in

b.eJ:;w -b..u 0 e le dvod..€.,4

F<:Nw a--rcl b,·cu e.d.. ..u 9, Ve.4A

End faces are polished.

E'3J

* wt, -t:-\,-\ -e.. 4-e..,u - .0...,y--e .fo,....wa rr-J.

biased _1o.rrs e_ h..t!> • &-! e,t PChet,u cw4

holes 0...'Y'le.. !Y"eC-0Mb t"v'\ e...d._ .

* Recombination produce large photons.

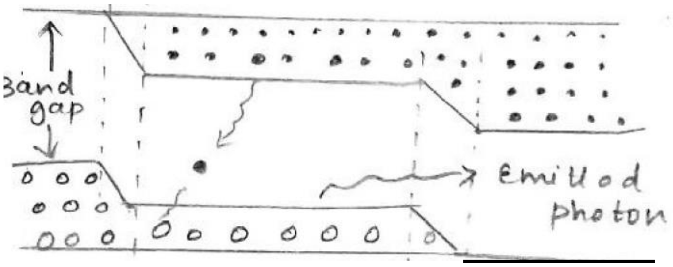
* --rhe_ b.iA,g jo VD ll:a. e ..u.

">'0 f'-t>,y-tiO'Y\..a.1 bu L-t,.. e. €..M.lli

efr e./t'c .

""'ttul- p\.,u, b, ru ecb eJ bcJ ti-.e

&tukeJ.. <2.-WU cu-ul_ - e.k- uke:l.



* :r b- ,,.-....Vd..u Covd:i 9<.U wtl,ve .fu""

* output power -.I..ll very high

Disadvantage :-

.#(Ve.Q dL-ff,-CL.4..U- -to .fu"llrY\ ]t.U\dis-k

"" ..u verry ½1"

*" di.-f'-c.u-.U- ..fo ..forrM. J<1.n cA:,,· tJ-11

* b ..a vO ½'k•

BASIC APPLICATIONS OF LASER

Ut A C.E. /\ L-LO 'f IN l; :
fl ml' o.Xt Th-0..A ivl
'u eo Y\ l YDl\ee- \Y'e\ l 1 5 of ror{L
6b W QY \L f I e.UL tl,Ll' (..Q. -lo
1 ye_a,} \:, _ wi 1h ,r1rn(A..\-
ClJ c) '-\b V''' 6\ rowq a:,\'OJ;"6-
Z\.Qxn v./\

L AJH?. Ct-APOI

(>)J Ov,S 71-1 l e-v vv\
m.e,,\ t, J t\ n l ,.e.,,- o+
\Iv() r\c... PI Q.U_ 'jlAA WI\\ \rn 'X

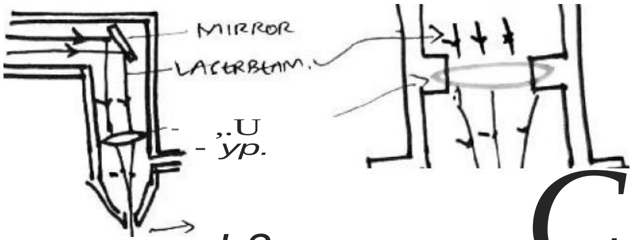
w 1 -rh tvu \A\ vv' cl U cx_ol cl.;"f
CU,LOL('
o El LA I<>' t-\\o'1i '''if'o
.1,) Co"--Ii ":!

l,A(tvG-t--P1"10)

vvelol, 1 joiV\1 { +wo (0-)
o "} vY\ oY'e_ rr·H:...,t01..-(p,.e.. \n +:>
c;,... S, ...Q. ,ntO5;,ng'\.e. o b
@ 'J: V1 we) cU Y\) -th..e b-eam
.> -rYl12 £..d c?f tvi.-e, twO
'P\u...te. lo l\l'\.€AY me)..\:;, pt), rlt
0-.#-.-d Cau_u_ th t:o fu.,u_ .
I<Z-IA.u.. No po re; 'o 1 0 ,-i of
I n, \) \./vY\ -n' , w Q..,\ y-o._:I. Id..;f I
(i/'V\,lcv.-- m-e.....W.c be_
vv.e..{ ot-e.d

LASER CUTTING

e_ \)..,-,ne,,, pie 0-J L Cu ..H1
'> ih.e. v eo'Yi?_ _-hon o.:f ti-le..
0 ...-t-e....--,a.1 0..t 0\... fo\h l 0--y foc.u_,J
o -tne lo.earn, The._ V oYi
Y),O\.t..t2Y-,ctl ,u Y oY,£>cl w;+v, the
h0lpof o... GI-1 jet_
G>Jt (O(_,\I\ b-e... don.e. a_;t room 1..en-,f.
(j) ClA.1-t i r1j pee ij V .
e> 0:-uh nj Oln.e\ too fj' QJvo .r o Y a...p1J.



===::::c:=!-3:::_:_______C
V\t.e.\ ol.i c...LLlt-l""/ c,. ,.pj
)11\,r\f-

,.OS c1 0 I.Anong,.

--<---A-.--A._..,,.. .-A--

1);f'5e '{Q ()"\-io.\ 59\.AD-hon "{ o'

Si N") plo._ t-\-0.'"< N'" on I c_ /\ oh OI") 1-

Definition:.of SHM:

Simple \-\et"frnoni c_ Moh at'" I

the mo oV1 in w\.,1c.."" e_

0-.cce-Q..Q'fo.non OJ 0\. body ! 5

d,.liQC-t.Q.t\ Poc'Po'!\iono.\ ·-k; tful.

d)9-,pl i;\ CJ2. fYU2v.t frro fY1 Oc_ :!1Xe_ 9

po1vrr o.Y'.d. IS Ql\1...Do.. s d11'1'ec:l-eq

'towo"f d"'-=> 1-rLe _yL'.X'ed po,"t CCf'{)

e..qv..., \1bY\1..AW") poe.1·ho,"i,

Derivation:

D 0 IA_C,JL

1 _ 1 Splo.C,Q_l'Y\Q,V" \- of V1 bmli

po'"<h ct!i o t o. ·,n Jo.vd- t s cla_fin

O_f;, -ti, e d.i.£·1-0.. CQ.._rnov_Qd \J 1-f,e_

Po'-"l'+icle f'aOm 1 s '(Y\.Qo...n pos1+1on

o_f ye.5(;,

1"'1 \.-ie n ttu- p°'1 H cJ.a_- p' mo ve.S

O-"fou..ncl.. c)--t c\e, -!"1en ot

0 \-1,,e \-:::€-"{ p ·nd i c.,u,lo..Y 0 1 \ /, byo.re

0-lon ('he oio. N\Q.n?'f-Y'f1,-u-rHw.y

\iei "f('\On 00 or \hQ f>!A.i" h c.)_!1 'p J v)

\.)._,V\A v-<i 1 \ 1hA- 010--hon e,1 · 1 v::i

CLUo pe ry1 oc.--ll C. . (la,) th.9. p 7-h c.Je.

— \ \:_rA'c.Q_ tha. c;:C. \JY\..O -b M'l +o VIb 'YO-e..

b e -h» eQ. '<1 1hi. po'rn ,; 'y o' '/\

· / o "l-f th.Q. p Cl.'i·h c..tii.. ' p, t,o pG> e..S

o 'fe,rok,lt\on 1 \/\ th.a.. foot t>f

r _0 'QJ w1\\ rOplilre ovu.

VQ"'!'tic\e. og,c..i \\o-:\,oV\.

Thus the distance OQ is

1.eY d O',..g, oti.J3pl o..CJl. (YU_n l- of +hR.

pv(1 +f cJe a. n d. f s, ol&. b\--e d_ 6j

\k e-r 'cf'.

=tf -tk P£A'"h c_lo_ '"N'I0VeS

Gd- W C,onSl Y C\ p"'l·+icb..'P' 'A ,t.o f ,t -Se c..onds, !k11 I

Toov,Y1 Z) I() v\. C,) ryc.u,\ o._y Po-1i0 °.f O\na le... \::,o,,.twB2."" ',(\s erV €.T'\ 1

"'{ v\ cl.11.A.S' ¢., 1 wi '\"\-:, UY'li'to'fm Yc;J.o c.i \LuJ r -.—._ e..pe,

\J-1 o..v,.Ji on \e,.._Y Vek c; W wI Tti

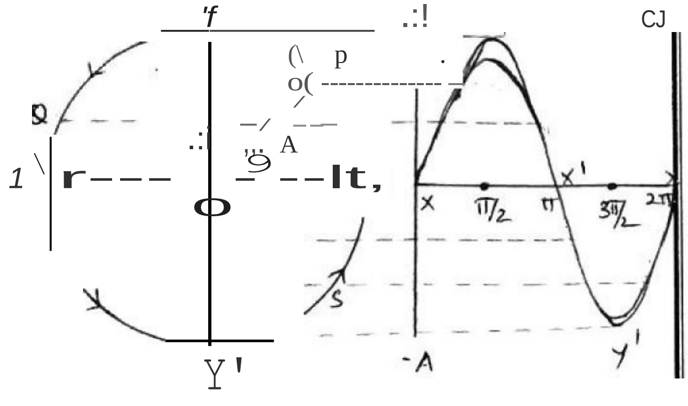
y e.s re. cl; to -t½e. CQ,.vi r rye. cf iho C,jnJ.a.._

:f Y eJe'f e.'(\ CQ_ 0 o,_g g h.o wn VV1 fi

'f .:! CJ f,uM 6 <StPO

O"(5-t-rr-eot

= L&PO,:: ():::.WC.



---0

sin θ (or) sin ωt -

OQ = y and OP = A.

sin ωt = y/A

(

y = A sin ωt

-§:_

1y<1s 29 /\ '(es pY eV\ _S. d 1 plo.u, rvtQ..'l'l

05- vi b'fo.HY\d P°"'•nc,.tQ_

(ii) Velocity:

V re-- '1\ r vf lk \/1\> n.h -"'<

\ I\ \1 \0 I C', – o -f, \0 \ 9, \ ho

rate of change of splacement

,'. \1 '\\ ' ,n '3

dh \-i h ih=· -4\'\ — '''' c.;(' 1..<x> 8rt

I,}- - d C f\Stn wt.)
c\t

\ : t,.. c,0 cos wt-
V'lt S \\ Veoc ·,1J

h,. C\t\ p'<

r '''-c. V; '(o.hl"(J po-'(-1-i c.\e. •

iit) Accek,"i0,.. n o.r,:-

A cc ele"<o.n on 5 \

\/1 Y<'.'-'1 """8 p0-r<·h Ce I I S c:;lo (',<\.Q.c\ J7

n S, l -yo.\- e.. o.f ch o..nQ_ a f

\(.9C,o 4 -

.: A cc e,.lp'C h ov"\ $= \frac{dv}{dt}$ — ⑤

su\:,sh\--u\--i"u etzi--,@ 9ih@

,Accete·w:hon :::. _0..- lA wcoswf:;)
oU:

$$\frac{d^2y}{dt^2} = A\omega(-\omega\sin\omega t)$$

$$= -\omega^2 A\sin\omega t$$

\-@

-n "< f>-Y vi\ S· \

o..ccd2.oyo.no'f'\ of vibrating
IA.·rh cJ.a_,

Unit - IV - Basic Quantum Mechanics.

Introduction:

'(V)i., o-J. ln.v1cl:t1ct \oL...~v, "' ,_, ,_ • '
. A . _L• L d "f""- .. — I'\ Sc.u.t.l"l LJ2. I .!.,
G,n:o<l ti/> / chV\tl- . C\µ-c.v\h.c.VV:, I'\ t/b b,Jkl' 1"1-\=\'1
..QJ.v e,'l'\D.'(\ Yn.a,tk.'",0.:, i e -tile-',\fl' "'0 ¾a. su. V\M'M'2-Y\ al P½261 .
jl.\."'OC.'t'f\S2.r.kJ- t.uQ,'(12_ no J.JJJ-,. 1..J7"1]""(- ,t l'lrCY\1 L')Vid.,..e.
,- ev,_"i) e><-\-te ,,., c,.u.J-. = ?cJ'/H J,,_ ff' h., J\,,,J , """-"', ,Jo,_.""
o,,,,). - . '11-,. ='4Jcb GM. ""f -.-.J <Sb " d'o.,<i=9."
'\k l,ua.:LA °15 cJ.-o C.0-.V"lno\.-- he.. olo.scY ovnvl umcl.tiv Sh,
u\\9' 6m 'tl>- 'fb,.. jJ- t:""c#_..1.:, Y p-oa:e, °t ak,m.
f>. nn+, 1;- ..)M't.v.h, 'hr.\JJ • 0 .,
- .$c."'t ct., ., P.-A·M • <Dl ,'w· c,...V t--'l· n, c.o-,.. o:wvA
,)-ur \n 1k. f\Yl,\r)a 1s :k> y, . YLQ.u9-
\o..H (7v'\ ., °'- b "-r-, 45 ? 0::, ., l,Joh ha.-m.ed o)-,. ,, Ol,a.t..h') VV\Q.JAW'\ ,,

LiM OY}..h 1s u,J. Y\ U)-
ill;. 1h.a. ?h0.'11lma.fl W't\1t.,h cJ c.ol P a.
h, e.?Lrbln 'v\ac.lr<.. b W,:cb'.,.n _,. 'y o\.o,e&J-.,c.. / l2mi!hion 1;
- 'Y • e_k- •
-.l(. e> m:u'..n 'tR..YlcL (\va'<'tb:)l.J
tele,. 1 &o.wco..R.f CP..,, OM. \oo½ UM Pc-MA·
t'J'NJX<Jj. · (J'(\ It,_ c.J.- ""J , "'''' "-i'''- 0°"',J? -liJ-)} c,,,.. h"""-
c.92<la- r)O'()- --J_0rrJ .

h) d Quantum Mechanics!

- 'r<14. Ch l ,e b_; \'Y\o oM
°'5 o J- w\v:cJ-\ au.. \Ja..bb.. (Tf" ½ M,c..'rOS<.o • uf-
Y't1Lc-h::u, 1a, G p-o lttt fC4YhJ!u... ci aA"r,r<t (., i .iJ:. f& Iv
e *\ oJ 'b-e.,\-i , ., rh..cz._ _ uJ. \"A...P A Y\ U:> Ca.Y\ Y\ or _lb
\ e.?LplNr1 1 n r-t.t.. ,-- e Mc Hon cf; e::\- ef o'fl \'{\ C-tY\ cJ..om ,

----------------------\,.±- '--

. 'fnL ?:11orYW.11A.- 't kla.t..h bod.,"! "(a.c,!,td::..,in 1 ?hoh:,efeJril-

-__,, erv-,rJ..»,loYl at 'X,- 'r ' @k• \µQ)U2- eO<f\Cl..l 6"2 t"lo.. ?la.nc.,\L. \f\ \'140

, taocluu:na 15 -\k m

-[s--=--nh-.,.)

Wh.o.rd.-,, '(\ '.:. 0 J 1/ C)..., ••••

h= P\c..vic..\<..1 '''\-- -=-{,L60 X.\O J ,

* 1b -- l<_'Ae;WY'i OS1;;, Q /Vs'() .S " amd mtt'Yl<Jd

1k. ''''' nj °f \'Y)oM/rl\ Y ch• v-' m(c..,obj;1L <.<.?o&d 0

tk r-'f(\, _ >'----

P hol--tm.> o.,,-,ol U rJ- \.)a,..tl, - C.'Dw:.1,J; d '1s LJc.1,•o"' °""J. rYJJl:uJ

' 0/11'\ol ? y'/t'c:.6., cf r Jiovi

ocl½ \<-Y)o\n 0-../4#°5,1,WSL vi o.-,,d,..

Pc.."l -d.e_. . •

Wo.. •

:J A Wc:ulL C)Y,CJlne-.k /4.AL to , .J ;_s .bP ou.1- OIIII,Y

o-)a,'{ oc((£_ - P> CQvv.,_ 'v'\o .Q.,u,kol d- a_ Yc..thCJ_, ?kaz.

CwiJ,_ 0,...Y\.11.0\-- (o,,,•((t.12..?. ½-.:'\.. w·

= h , °'-= - u."'- SJ, ,o.J- ½•ix,.,,u. , boa .;l,,

A, ""•J ;:> , ti - '),,. f\,a,,e S o,,,,,l'.kh».11 :r:.: •

1k. Pho-noYYl.Q.t)c;_ Cl J)nh,'<.,f'i.e"' J ?o\o.yi £AL011

1k, kJQ.\l\<J-1 Lo 'qy· Y'1rX1L w Q. 1'vne, cJ-1&.a, .5Rm.e..

r>o-f>1btof) •

T\.- \b \J cfaarf lu.,o or fc.,,rh'® Ca.f\nol-- o -ML

b,.'<i'JQ, 1po&.l,:o d- It. f.,o.""2- \:("""2.• .So ow,. \,,,,. h, urn k}-

Yo.ch d;CNI J.;k •

P kk:

* A i ck · k!eJ. cJ- '('(lQ, J.1.J|·t\,fR.. Po·,11 a,.,,J

J} b ''' \- - C/'N)__ fb hi q_ tkv . A- PCvYh't4_

w:l w 1)- cuul1il1-cf--e.JOM-d JX c.J,Q_r, Ji

5\ &v'(),

L , *, A fc,,.h<.h- ·u, c,bc.Ja;,1},J ½ !Yb.vs'.!:"! \J.,,J,,c.;L,/-;

'Mo(Y\.iltAf\.L'('() p (l.K".J f'..JYIQ y

* Spectra of black body radiation, Compton effect, Photo electric effect etc. could not be explained on wave nature of radiation.

These Phenemena established that radiant energy interacts with matter in the form of "Photons or quanta". Therefore, Planck's quantum theory came to conclude that radiation behaves like Particles.

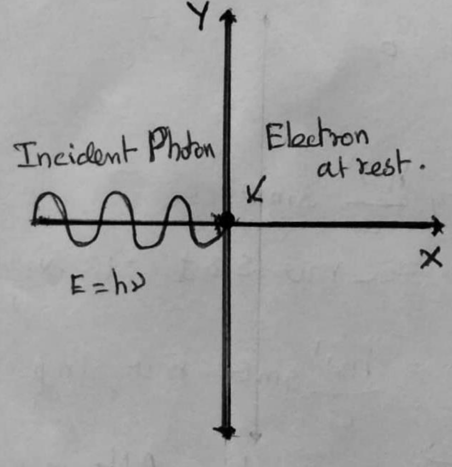
* Thus, radiation sometimes behaves as a wave and at some other times as a Particle. Now, wave-Particle duality of radiation is universally accepted.

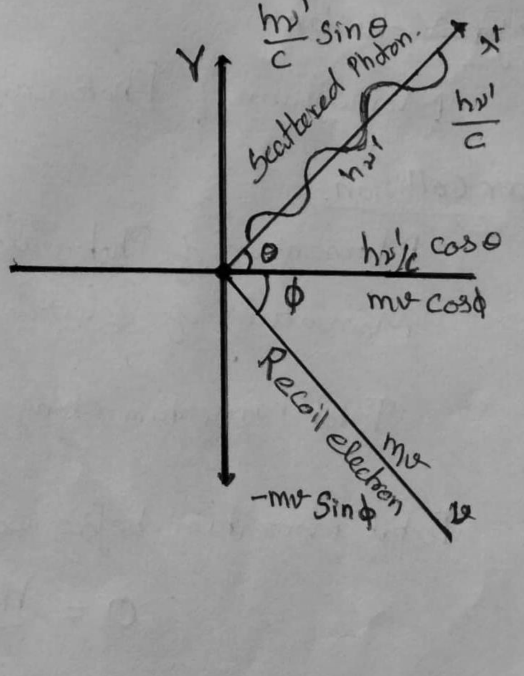
— x —

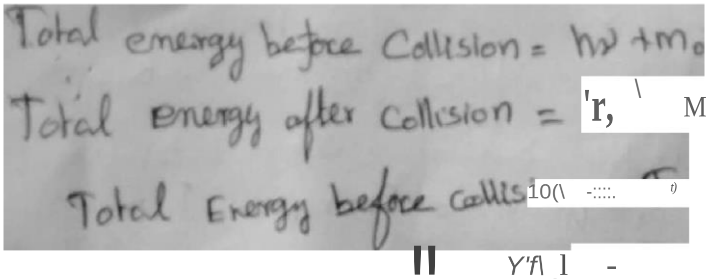
Compton Effect:

Statement:

"When a beam of 'X'-rays is scattered by a Substance of low atomic number, the scattered X-ray radiation consists of two Components. one component has the same wavelength 'λ' as the Incident ray and the other Component has a slightly longer wavelength 'λ''. The change in the wavelength of scattered X-rays is known as Compton shift. The Pheromenon is called Compton Effect."







Total energy before Collision = $h\nu + m_0$ C.

Total Energy after Collision = 'r, \ MC.

Total Energy before Collis 10(\ -::::. t) , (: ifJ r Collision

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$mc^2 = h\nu$ -V_ '1"1'\(.l.

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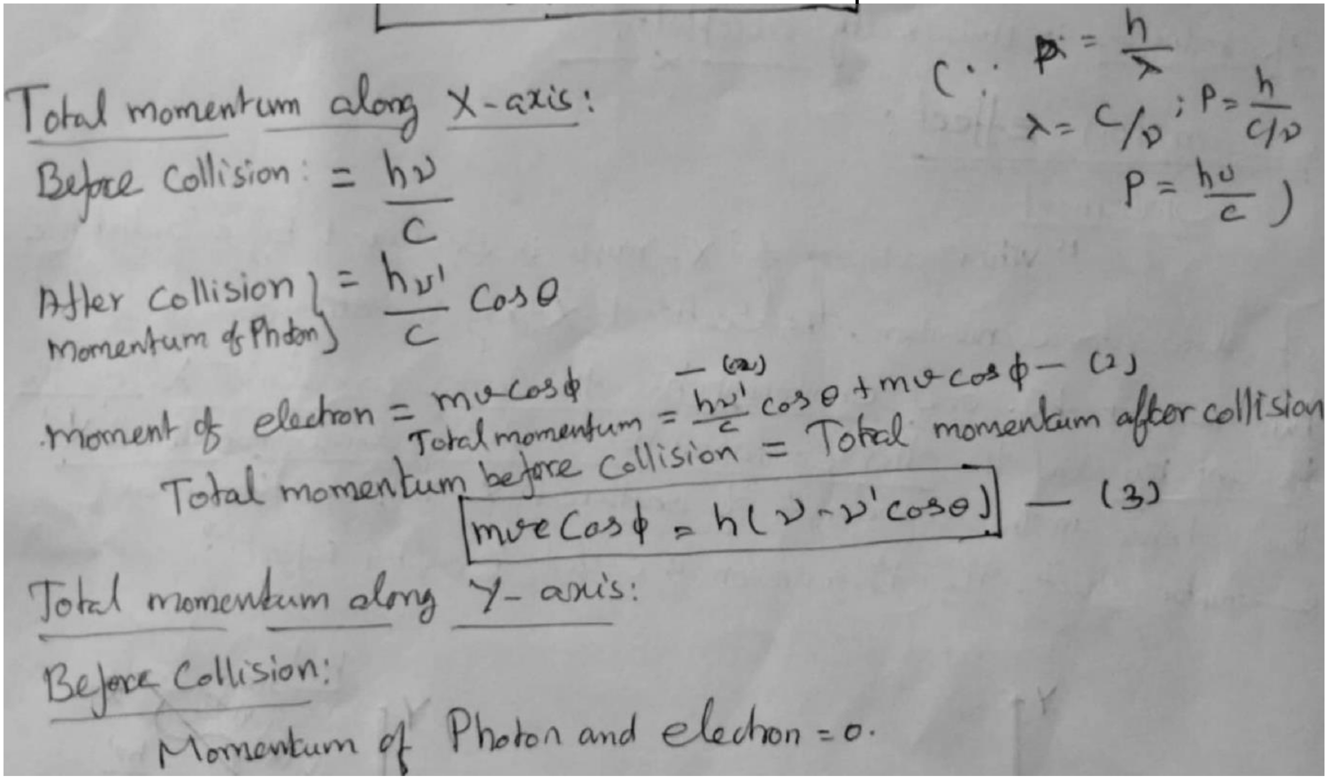

Total momentum along X-axis:

(∵ $p = \frac{h}{\lambda}$

$\lambda = c/\nu$; $p = \frac{h}{c/\nu}$

$p = \frac{h\nu}{c}$)

Before Collision : $= \frac{h\nu}{c}$

After collision Momentum of Photon } $= \frac{h\nu'}{c} \cos\theta$

moment of electron $= mv\cos\phi$ — (2)

Total momentum $= \frac{h\nu'}{c}\cos\theta + mv\cos\phi$ — (2)

Total momentum before Collision = Total momentum after collision

$$\boxed{mv\cos\phi = h(\nu - \nu'\cos\theta)} \quad \text{— (3)}$$

Total momentum along Y-axis:

Before Collision:

Momentum of Photon and electron = 0.

After C 'jlor"J:

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-r\{\-..,...-C---$-\Y\-=- -::-'.!,_ "' i11 l', Squaring eqn(3) and eqn(5) and then adding, we get

$$(mvc\cos\phi)^2+(mvc\sin\phi)^2 = h^2(\nu-\nu'\cos\theta)^2+(h\nu'\sin\theta)^2 \quad \text{—— (6)}$$

L.H.S of eqn(6)

$$= m^2v^2c^2\cos^2\phi + m^2v^2c^2\sin^2\phi$$

$$= m^2v^2c^2(\cos^2\phi+\sin^2\phi)$$

$$= m^2v^2c^2 \qquad [\because \sin^2\phi+\cos^2\phi = 1]$$

R.H.S of eqn(6)

$$= \underline{\underline{h}}^2(\nu^2-2\nu\nu'\cos\theta+\nu'^2\cos^2\theta)+\underline{\underline{h}}^2\nu'^2\sin^2\theta$$

$$= h^2[\nu^2-2\nu\nu'\cos\theta+\underline{\underline{\nu'^2}}\cos^2\theta+\underline{\underline{\nu'^2}}\sin^2\theta]$$

$$= h^2[\nu^2-2\nu\nu'\cos\theta+\nu'^2(\cos^2\theta+\sin^2\theta)]$$

$$(\because \cos^2\theta+\sin^2\theta = 1)$$

$$= h^2[\nu^2-2\nu\nu'\cos\theta+\nu'^2]$$

L.H.S = R.H.S

$$\boxed{m^2v^2c^2 = h^2(\nu^2-2\nu\nu'\cos\theta+\nu'^2)} \quad \text{—— (7)}$$

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$$\boxed{m^2c^4 =}$$

Subtracting eqn(7) from eqn (9), we get

$$m^2c^4 - m^2v^2c^2 = h^2(\nu^2-2\nu\nu'+\nu'^2)+2h(\nu-\nu')m_oc^2+m_o^2c^4$$

$$-h^2(\nu^2-2\nu\nu'\cos\theta+\nu'^2)$$

$$m^2c^2(c^2-v^2) = \cancel{h^2\nu^2}-2h^2\nu\nu'+\cancel{h^2\nu'^2}+2h(\nu-\nu')m_oc^2+m_o^2c^4$$

$$-\cancel{h^2\nu^2}-2h^2\nu\nu'\cos\theta-\cancel{h^2\nu'^2}$$

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From the theory of Relativity, the variation of mass with velocity is given by.

$$m = \frac{m_0}{\sqrt{1-v^2/c^2}}$$ - QI)

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$$m^2(c^2-v^2) = m_0^2c^2$$

multiplying "c^2" on both sides, we have

$$\boxed{m^2c^2(c^2-v^2) = m_0^2c^4} \quad \text{——} \quad (12)$$

Substituting eqn(12) in eqn(10), we get

$$\cancel{m_0^2c^4} = -2h^2\nu\nu'(1-\cos\theta) + 2h(\nu-\nu')m_0c^2 + \cancel{m_0^2c^4}$$

$$\underline{2}h(\nu-\nu')m_0c^2 = 2h^2\nu\nu'(1-\cos\Theta)$$

$$(\nu-\nu')m_0c^2 = \frac{\cancel{2}h^{\cancel{2}}\nu\nu'}{\cancel{2}\cancel{h}}(1-\cos\theta)$$

$$\frac{(\nu-\nu')}{\nu\nu'} = \frac{h}{m_0c^2}(1-\cos\theta)$$

$$\frac{\cancel{\nu}}{\cancel{\nu}\nu'} - \frac{\cancel{\nu'}}{\nu\cancel{\nu'}} = \frac{h}{m_0c^2}(1-\cos\theta)$$

$$\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0c^2}(1-\cos\theta) \quad —(13)$$

multiplying 'c' on both sides of eqn(13), we get

$$\frac{c}{\nu'} - \frac{c}{\nu} = \frac{h\cancel{c}}{m_0c^{\cancel{2}}}(1-\cos\theta) \qquad \left(\because \frac{c}{\nu} = \lambda,\ c/\nu' = \lambda'\right)$$

$$\lambda' - \lambda = \frac{h}{m_0c}(1-\cos\theta)$$

$$\boxed{d\lambda = \frac{h}{m_0c}(1-\cos\theta)}$$

Therefore, the change in wavelength is given by

$$\boxed{d\lambda = \frac{h}{m_o c}(1-\cos\theta)}$$

It is found the change in wavelength ($d\lambda$) does not depend on the wavelength of the incident radiation and the scattering substance. But it depends only on the angle (θ)

Case(i): when $\theta = 0$, then

$$d\lambda = \frac{h}{m_o c}(1-\cos\theta)$$

($\cos 0 =$)

$$= \frac{h}{m_o c}(1-1)$$

$$\boxed{d\lambda = 0}$$

(Along the incident direction, there is no change in wavelength).

Cos (ii) When $\theta = 90°$, Then

$$= \frac{h}{\text{MoC..}}(1-\cos 90°)$$

(…os $90 = 0$)

$$d\lambda = \frac{h}{m_o c}(1-0)$$

$$\boxed{d\lambda = \frac{h}{m_o c}} \qquad \boxed{d\lambda = 0.0243\ \text{Å}}$$

This difference in wavelength is known as Compton wavelength of electron

Case (iii): When $\theta = 180°$, Then.

$$d\lambda = \frac{h}{m_o c}(1-\cos 180°)$$

($\because \cos 180° = -1$)

$$d\lambda = \frac{h}{m_o c}(1-(-1))$$

$$\boxed{d\lambda = \frac{2h}{m_o c}}$$

$$\boxed{d\lambda = 0.0486\ \text{Å}}$$

Thus, The change in wavelength is maxit'l'\'-II'\IJ ::.. 1 o"

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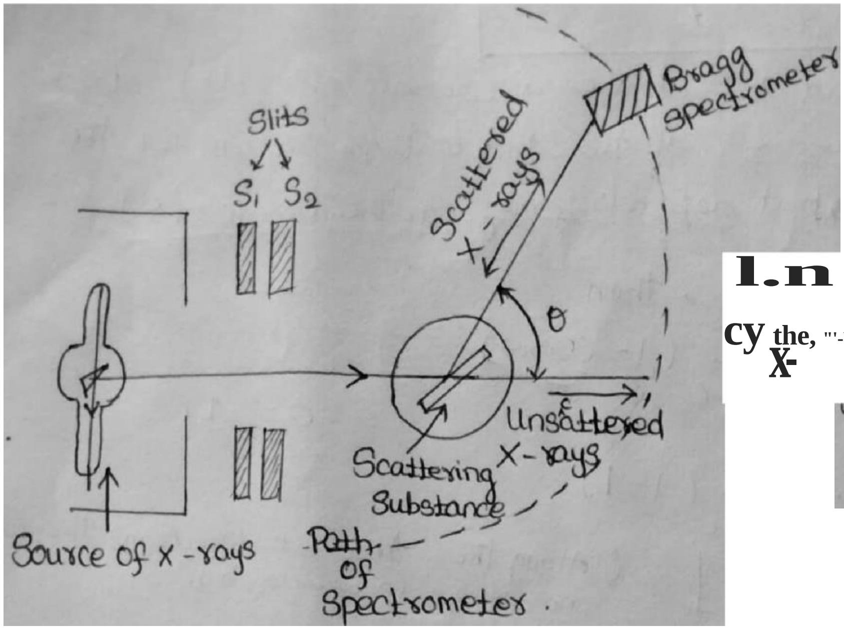

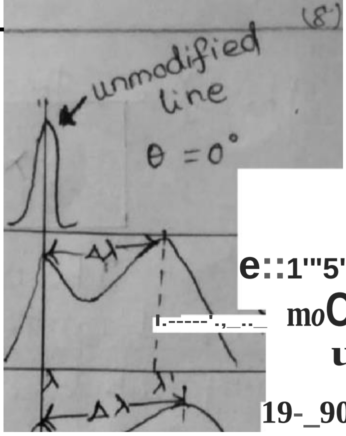

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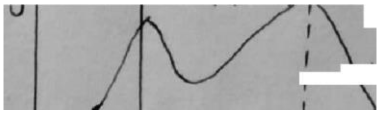

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$$d\lambda = \frac{h}{M_0 c}(1-\cos\theta$$

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$$\lambda = \frac{hc}{mc^2}$$

$$\boxed{\lambda = \frac{h}{mc}} \text{ —— (3)}$$

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$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$$

De-Broglie's wavelength in terms of Energy is

$$\boxed{\lambda = \frac{h}{\sqrt{2mE}}}$$

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$$h = 6.625 \times 10^{-34}\ Js,\ m = 9.1 \times 10^{-31}\ kg,\ e = 1.6 \times 10^{-19}\ C$$

$$\boxed{\lambda = \frac{12.25}{\sqrt{V}} \times 10^{-10}\ \text{metre}}$$

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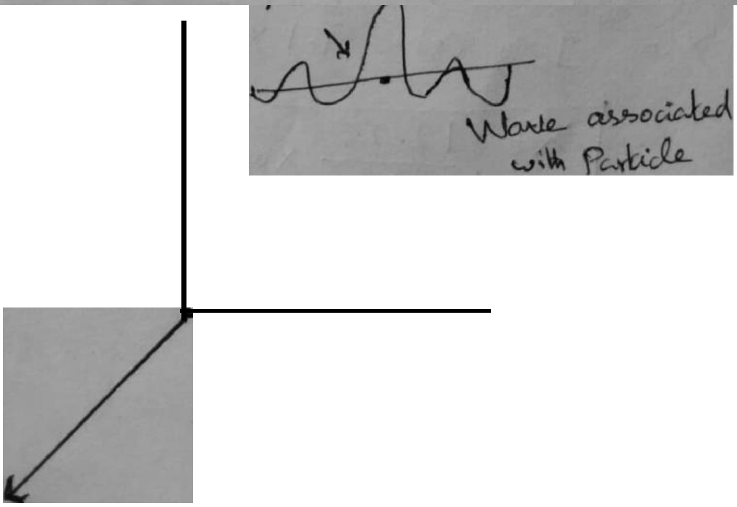
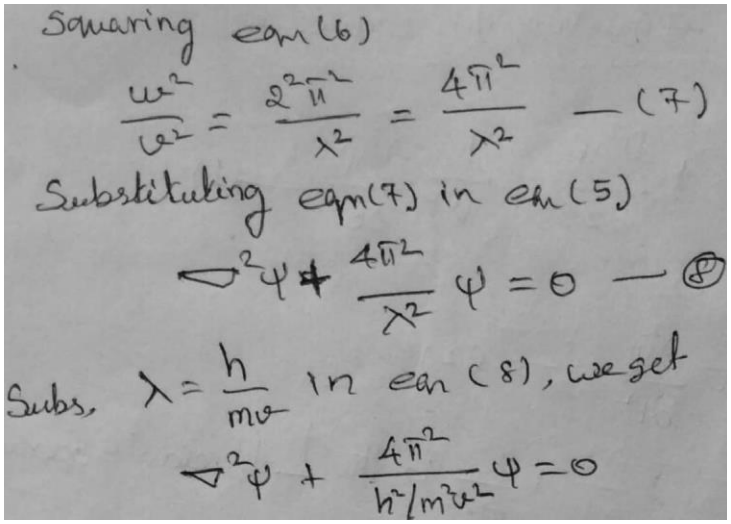
Schrodinger Time Independent wave equation:

Consider a wave associated with a moving Particle. Let x, y, z be the Coordinates of the particle and 'ψ' wavefunction for de-Broglie's waves at any given instant of time 't'.





Squaring eqn (6)

$$\frac{\omega^2}{v^2} = \frac{2^2\pi^2}{\lambda^2} = \frac{4\pi^2}{\lambda^2} \quad —(7)$$

Substituting eqn(7) in eqn (5)

$$\nabla^2\psi + \frac{4\pi^2}{\lambda^2}\psi = 0 \quad —⑧$$

Subs, $\lambda = \frac{h}{mv}$ in eqn (8), we get

$$\nabla^2\psi + \frac{4\pi^2}{h^2/m^2v^2}\psi = 0$$

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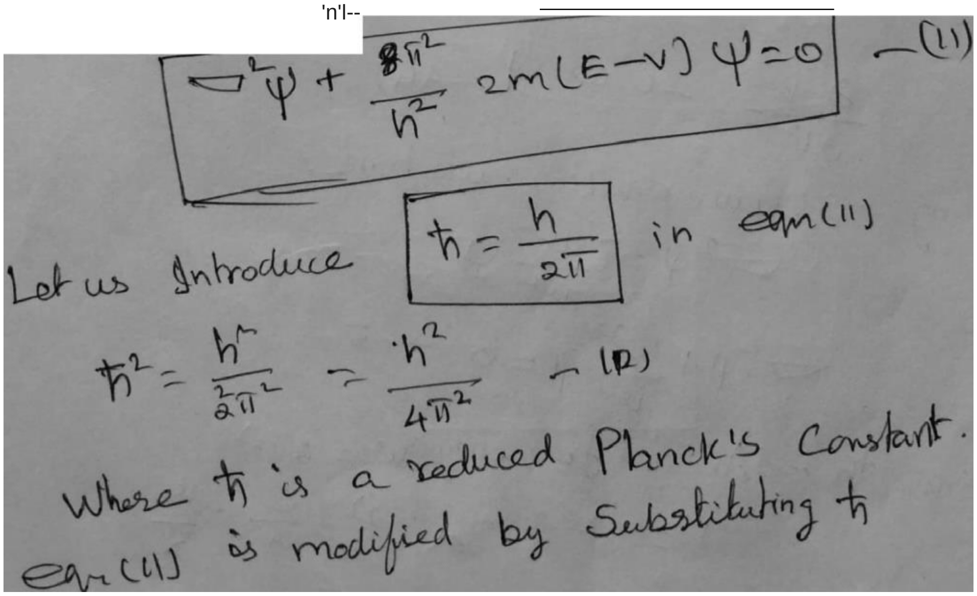

$$\boxed{\nabla^2\psi + \frac{8\pi^2}{h^2} 2m(E-V)\psi = 0} \quad —(11)$$

Let us Introduce $\boxed{\hbar = \frac{h}{2\pi}}$ in eqn(11)

$$\hbar^2 = \frac{h^2}{2^2\pi^2} = \frac{h^2}{4\pi^2} \quad —(12)$$

Where $\hbar$ is a reduced Planck's Constant. eqn(11) is modified by Substituting $\hbar$

. lu\

The classical differential equation for wave motion is given by

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}=\frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \quad (1)$$

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$$\psi(x,y,z,t)=\psi_0(x,y,z)\,e^{-i\omega t}$$

$$\psi=\psi_0\,e^{-i\omega t} \quad (3)$$

Differentiating the eqn(3) with respect to 't', we have

$$\frac{\partial\psi}{\partial t}=-i\omega\,\psi_0\,e^{-i\omega t}$$

Again Differentiating w.r.t time 't'

$$\frac{\partial^2\psi}{\partial t^2}=(-i\omega)(-i\omega)\,\psi_0\,e^{-i\omega t}$$

$(\because \psi=\psi_0 e^{-i\omega t})$

$i^2=-1$

$$\frac{\partial^2\psi}{\partial t^2}=-i^2\omega\,\psi \quad (4)$$

Substituting eqn (4) in eqn(2), we have.

$$\nabla^2\psi=-\frac{\omega^2}{v^2}\psi$$

$$\nabla^2\psi+\frac{\omega^2}{v^2}\psi=0 \quad 5$$

We Know that Angular frequency $\omega=2\pi\nu$

$\nu=$ frequency ; $\nu=\frac{c}{\lambda}=\frac{v}{\lambda}$

$$\omega=\frac{2\pi v}{\lambda} \;;\; \frac{\omega}{v}=\frac{2\pi}{\lambda} \quad (6)$$

$$\nabla^2\psi + \frac{m}{h^2/8\pi^2}(E-V)\psi = 0$$

$$\nabla^2\psi + \frac{m}{h^2/4\pi^2\times 2}(E-V)\psi = 0$$

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Schrodinger Time Dependent wave equation:

Schrodinger Time dependent wave equation is derived from Schrodinger's time independent wave equation.

The Solution of classical differential equation of wave motion is given by,

$$\psi = \psi_0 e^{-i\omega t} \quad —(1)$$

Differentiating eqn (1) w.r.t to 'time' t'

$$\frac{\partial\psi}{\partial t} = -i\omega\psi_0 e^{-i\omega t} \quad —(2) \quad \because \psi = \psi_0 e^{-i\omega t}$$

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$$\frac{i\partial\psi}{\partial t} = -i \times i \frac{E}{\hbar}\psi \qquad (\because i^2 = -1)$$

$$\boxed{i\hbar\frac{\partial\psi}{\partial t} = E\psi} \qquad -(5)$$

Schrodinger time Independent Eqn is

$$\boxed{-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi} \qquad -(6)$$

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$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = i\hbar\frac{\partial\psi}{\partial t} \qquad -(7)$$

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Physical Significance of wave function 'ψ':

1. The Variable quantity which describes de-Broglie wave is called "wave function 'ψ'".

2. It connects the Particle nature and its associated wave nature statistically.

3. The wave function associated with the moving Particle at a Particular Instant of time and at a Particular point in space is related to the Probability of finding the particle at that Instant and at that Point.

4. The Probability 0 corresponds to the certainty of not finding the particle and '1' corresponds to certainty of finding the particle.

i.e $\iiint \psi^* \psi = 1$. if Particle is Present

$\psi^* \psi = 0$. if Particle is not Present.
($\psi^* \rightarrow$ complex conjugate of 'ψ')

5. The Probability of finding a particle at a particular region must be real and positive, but the wave function 'ψ' is in general Complex quantity.

Motion of a Free Particle:

Letus consider electrons probagating freely in Space in the Positive x- direction and not acted upon by any force. Their Potential Energy is zero.

$\nabla^2\psi + \frac{8\pi^2 m}{\hbar^2}(E-V)\psi = 0$ reduces to

$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m E}{\hbar^2}\psi = 0$; taking $k^2 = \frac{8\pi^2 m E}{\hbar^2}$ weget

$\boxed{\frac{d^2\psi}{dx^2} + k^2\psi = 0}$ The general Solution of this eqn is

$\psi(x) = A e^{ikx} + B e^{-ikx}$ (A & B are constant)

The wave only Propagate in Positive x- direction. we get,

$\psi(x,t) = A e^{ikx} e^{-i\omega t}$

The allowed energy values form a continuum and are given by

$\boxed{E = \frac{\hbar^2 k^2}{8\pi^2 m}}$ $\boxed{E \propto k^2}$

rc;,"('lck \t1 =-lv\ \f\l[t2 J bw •.

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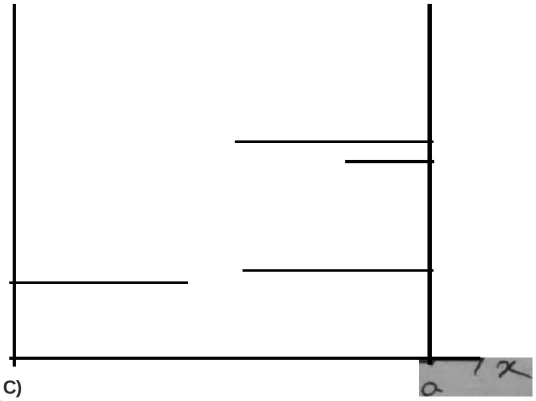

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'7k tp,_,l-,;J,,.. 6'Mbl-- c.,-.,.,.,_ J f ---\k 6>)(_, Als,o))- ca.vi

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&.<b.s j , '"-£ ,k'- \"' e,,,,.cl ,-„") v.iL

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$$\frac{d^2\psi}{dx^2} + k^2\psi = 0 \quad \text{— (3)}$$

J &\ ("\ eG,M l ?)) ..

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b o,Jj.,,ns,

C '<Mt_\) v o o1- o-

Af+¥o 16 n 6 CA), v->-"-'ocw-L

D -:;:;. A .s,n D + i c.o 0

∵ sin 0 = 0
cos 0 = 1

= 0 +- ,c;._

@ o]

Hence

:'J n ci\)

o cJ-- ?-_::.a.

0 ::: A Sh'\ -k +D

A s,n -tz -=o

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i,t Z€x't> Y\.. '1k t...vo.ML rju.n n '2J!JrO \..,

rt .. 'A' c.StioJJ nol-- ha- 2-RfD •

$\therefore \sin ka = 0.$

$\sin ka$ is '0' only when ka takes the value of $n\pi$

$$ka = n\pi$$

$$n = 1, 2, 3 \ldots$$

$$k = \frac{n\pi}{a} \quad \text{— (5)}$$

$$k^2 = \frac{n^2\pi^2}{a^2} \quad \text{— (6)}$$

we know that $k^2 = \frac{2mE}{\hbar^2} = \frac{2mE}{\left(h^2/2\pi^2\right)} = \frac{2mE \times 4\pi^2}{h^2}$ $\left(\because \hbar = h/2\pi\right)$

$$k^2 = \frac{8\pi^2 mE}{h^2} \quad \text{— (7)}$$

Equating eqn (6) and (7)

$$\frac{n^2\cancel{\pi^2}}{a^2} = \frac{8\cancel{\pi^2} mE}{h^2}$$

$$\boxed{E_n = \frac{n^2h^2}{8ma^2}} \quad \text{— (8)}$$

Subsituting eqn (5) in eqn(4), we have.

$$\boxed{\psi_n(x) = A \sin \frac{n\pi x}{a}} \quad \text{— (9)}$$

Here $n = 1, 2, 3$

For each value of n, there is an "energy level"

Each Value of E_n is known as Energy Eigen Value and the Corresponding ψ_n is Called as eigen function.

Normalisation of wave function:

The constant 'A' is determined by normalisation of wave function as follows.

Probability density is given by $\psi^*\psi$

w.r.t $\psi_n(x) = A \sin\frac{n\pi x}{a}$

$\psi^*\psi =$ *A* ·1 Y\ L **A Si** $n\pi x$ -

[∵ =- 'V W:._jw,d:.;,,.. '):"er_Q (..nc¾- c,,,.ph: j]

'f '-t - A'L .5111 L n!J -1\O)

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- 1.-.l, k <lkoot. 1 -ct' ½,

j "\' j.. 'V d')I_ cc' :L -(._l I)

Substitute eqn (10) in eqn (11)

$$\int_0^a A^2 \sin^2\left(\frac{n\pi x}{a}\right) dx = 1$$

r. 'l. 1.- c 2. 07
L Sr12 o-:. j

$$\frac{A^2}{2}\left[\int_0^a dx - \int_0^a \cos\left(\frac{2n\pi x}{a}\right) dx\right] = 1$$

$\left[\because \cos nx = \frac{\sin nx}{n}\right]$

$$\frac{A^2}{2}\left[[x]_0^a - \left[\frac{\sin\left(\frac{2\pi n x}{a}\right)}{\frac{2n\pi}{a}}\right]_0^a\right] = 1$$

0

'\k 5,.,cm, """1; 11-L 1\ \.a. = aJ- b\ri _t.,.JJ

A,z_ c..

- -)0 l.

$\frac{A^2 a}{2} =$

$A^2 = 2/a$

$$\boxed{A = \sqrt{2/a}} \quad \text{—(12)}$$

on Substituting eqn (12) in eqn (9), we have

$$\boxed{\psi_n = \sqrt{2/a}\ \sin\frac{n\pi x}{a}} \quad \text{——} (13)$$

This expression (13) is known as "normalised eigen function"

From eqn (8) and (13), the following cases can be taken and they explain the motion of electron in one dimensional box.

Case (i): $\boxed{n=1}$

$$E_1 = \frac{h^2}{8ma^2} \;; \quad \psi_1(x) = \sqrt{2/a}\ \sin\left(\frac{\pi x}{a}\right)$$

$\psi_1(x)$ is maximum at exactly middle of the box.

Case (ii): $\boxed{n=2}$

$$E_2 = \frac{4h^2}{8ma^2} = 4E_1 \;; \quad \psi_2(x) = \sqrt{2/a}\ \sin\left(\frac{2\pi x}{a}\right)$$

$\psi_2(x)$ is maximum at quarter distance from either sides of the bo

Case (iii) $\boxed{n=3}$

$$E_3 = \frac{9h^2}{8ma^2} = 9E_1 \;; \quad \psi_3(x) = \sqrt{2/a}\ \sin\left(\frac{3\pi x}{a}\right)$$

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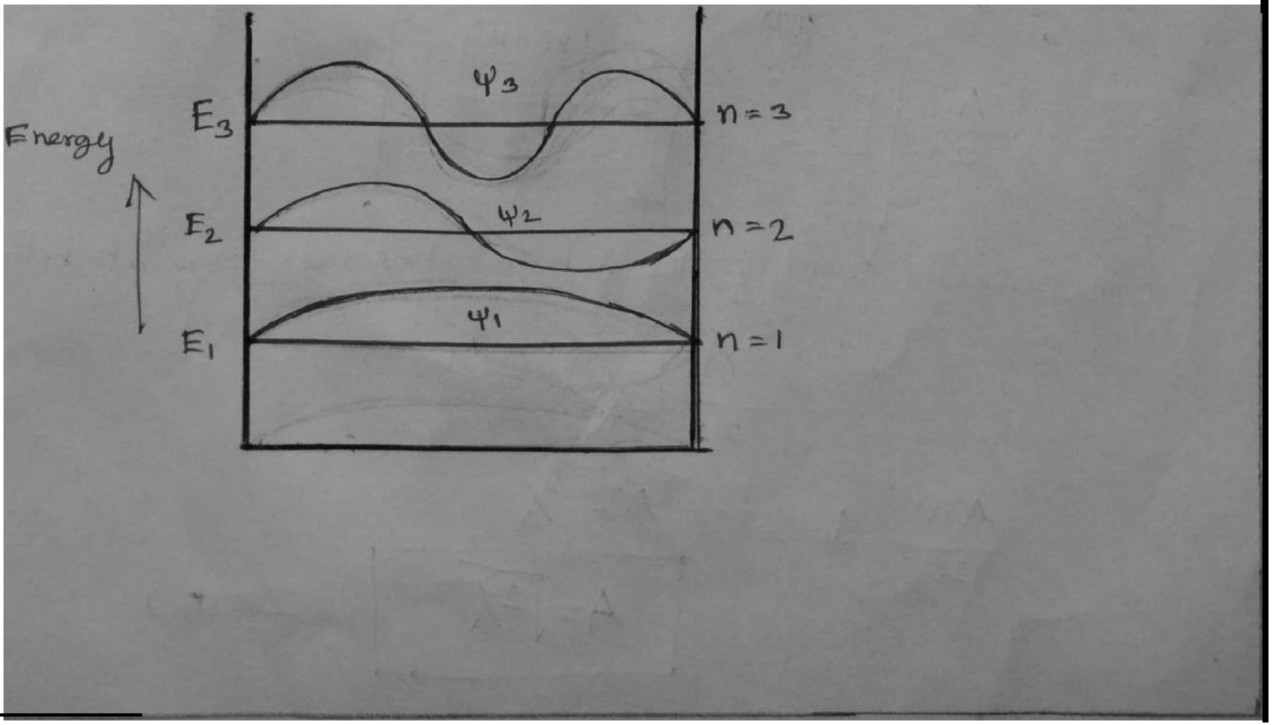

(23)

Extension to Two Dimension (2D Boxes)

In a Two dimensional Potential well, The Particle.

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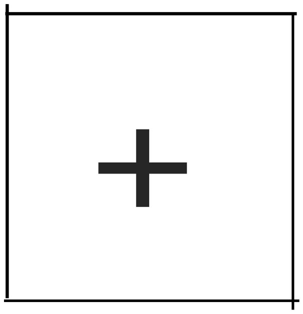

..

Energy of the Particle $E = E_{n_x} + E_{n_y}$

$$E_{n_{xy}} = \frac{n^2h^2}{8ma^2} + \frac{n^2h^2}{}$$

h'\b1... u

If $a=b$

$E_{n_x n_y} =$ 'n'l- -- L_Y1?C. Y) Q.,'2.-

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J; nte\'\S1or'1J u->_aR)_ u wn 'Ctb

tkn J2-/0 SiYl _____J f/a_ s,,, t J

1

- wq_ fk Sin);__ fol :)

$$\therefore \psi_{n_x n_y} = \sqrt{\frac{4}{ab}} \sin\left(\frac{n_x \pi x}{a}\right) \times \sin\left(\frac{n_y \pi y}{b}\right)$$

example:

$n_x = 1, n_y = 2$; similarly $n_x = 2, n_y = 1$

$$\boxed{E_{12} = E_{21} = \frac{5h^2}{8ma^2}}$$

$$\boxed{\begin{array}{l} \psi_{12} = \sqrt{4/ab}\, \sin\frac{\pi x}{a} \sin\frac{2\pi y}{b} \\ \psi_{21} = \sqrt{4/ab}\, \sin\frac{2\pi x}{a} \sin\frac{\pi y}{a} \end{array}}$$

We understand that several combinations of the two quantum numbers (n_x & n_y) lead to different eigen value & eigen functions.

Particle in Three Dimensional Box:

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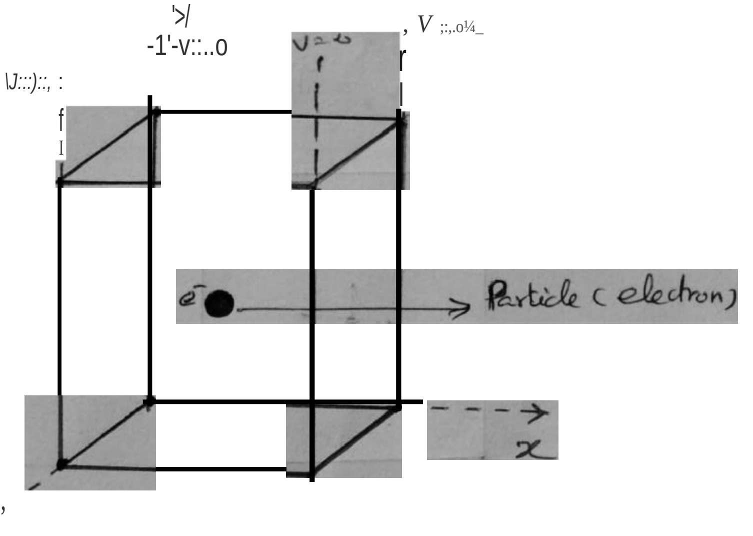
\/\- h\,.t> h, ,, ht- C6 Y'CL-!> pondl1 .\-t9 1'l.L u:>oYcJ;. n&vtL

0. Y) vne½ ?1-/ 0-J-1-- .



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EY\... n nz.-. - -- -- +---

r'1 'i? M \,'2- g--m e..'L

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n')Ln1 n:2-.- Es'r()a..,'l.-

Corresponding normalised wave function

$$\psi_{n_x n_y n_z} = \sqrt{2/a}\,\sin\left(\frac{n_x \pi x}{a}\right) \times \sqrt{2/b}\,\sin\left(\frac{n_y \pi y}{b}\right) \times \sqrt{2/c}\,\sin\left(\frac{n_z \pi z}{c}\right)$$

$$\therefore \boxed{\psi_{n_x n_y n_z} = \sqrt{8/abc}\,\sin\left(\frac{n_x \pi x}{a}\right)\sin\left(\frac{n_y \pi y}{b}\right)\sin\left(\frac{n_z \pi z}{c}\right)} \quad -(2)$$

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t_'<\,.,YI Jnz.) 'b e.ir vcltl. -

ei

'X 'VY\

(25)

$n_x = 1, n_y = 1, n_z = 2$

$n_x^2 + n_y^2 + n_z^2 = 1^2 + 1^2 + 2^2 = 6$ similarly, $n_x = 1, n_y = 2, n_z = 1$

and for $n_x = 2, n_y = 1, n_z = 1$ we have '6'

E I\'.L - E I - $E_{211} = \frac{6h^2}{8ma^2}$ — (3)

1k eo'("(LS F\,, Wave function is written as

Yn2-- ::- '{i3 Siy\ - I tr o1n e. \' U $ i '()'L\:I -:z_.

0... -;- C...

$$\psi_{121} = \sqrt{8/a^3}\, \sin\frac{\pi x}{a} \sin\frac{2\pi y}{b} \sin\frac{\pi z}{c}$$ — (4)

$$\psi_{211} = \sqrt{8/a^3}\, \sin\frac{2\pi x}{a} \sin\frac{\pi y}{b} \sin\frac{\pi z}{c}$$

1c,n'1 l) l '(c&. '15

'(Y-) ,·W- -- 'M€.- 5 \hOJA.2- h«J

.J– "" &.ch 0- ,a,\ak_ t Ja..J .. ea.,lW.

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No(\ - .sili

Wha Y) W}

Eigen value, Siit-h °'- s cJ.Qa) "" '1of\- J–L sb.k •

Suppose: $n_x = 2, n_y = 2, n_z = 2$

Then, $E_{222} = \frac{12h^2}{8ma^2}$ and

$$\psi_{222} = \sqrt{\frac{8}{a^3}}\, \sin\frac{2\pi x}{a} \sin\frac{2\pi y}{b} \sin\frac{2\pi z}{c}$$

— x —

'f1o n£. : (.12.r,,)

..• r 1h <tk_ r •

';;(_ anJ 0<- + ohc

p (11_) =- NJ'L d?L =- bn'V. ==-)d ,L.

Probability density is maximum when

$$\frac{n\pi x}{a} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$$

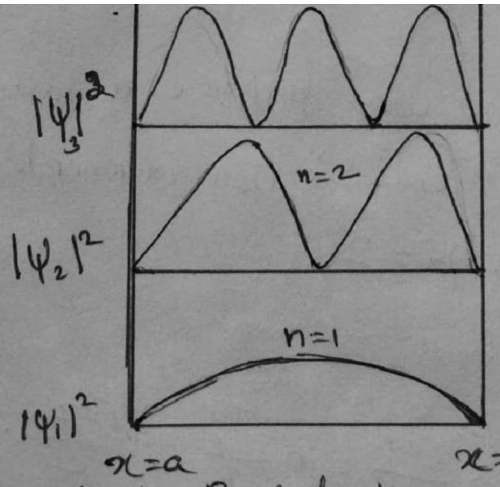
$$x = \frac{a}{2n}, \frac{3a}{2n}, \frac{5a}{2n}.$$

* $n=1$, $x = \frac{a}{2}$. The Particle most likely to be in the middle of the box (because $|\psi_1|^2$ is maximum there)

* For $n=2$, $x = \frac{a}{4}$ and $\frac{3a}{4}$, The Particle most likely to be at $\frac{a}{4}$ and $\frac{3a}{4}$ and never found in the middle of the box because $|\psi_2|^2$ is zero There.

* For $n=3$; The most likely position of Particle are $x = \frac{a}{6}, \frac{3a}{6}, \frac{5a}{6}$

Y\-;:_ 3,



While classical mechanics Predicts the same Probability for the Particle being anywhere in the box. Quantum mechanics Predicts that the Probability is different at different Points and There are Points (nodes) where the Particle is never found.

Correspondance Principle:

Quantum mechanics is highly successful in describing microscoping entities like atoms and elementary Particles. But, macroscopic system, like tennis ball, automobile etc, are accurately described by classical mechanics.

Bohr's Correspondance Principle bridges the gap between the Classical mechanics and Quantum mechanics. it removes The apparent discontinuity between these two.

Statement:

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— ∝ —

Unit - V

APPLItD 6JUAN<ftJH MECHANICS

5.1. THE HARMONIC OSCILLATOR :

Any Oscillation System For which u; 0\t ..Qc_k-l'j c-{"""\- DC"'.o..\ CO -tw r""{""..Qj, n9 \"\.O..<:P-\-t t\A.t. p\.a. t Q_Q. \\Jtd. a...n \,.,. Oscillator.

A u_\.US() ' . 0... PCP'c_\..-9.. 0,...'\"\.O...C .\/'-9-c\ \::.0 a.. .Q?'·n-D9 . <:)CT ma.. °'\J" O o._\--C) Q..("'.d.. (""n \.9u.J,.. c._o..C'\ C>..C2-o c.. b.od GS,. 0. \A ('n.Q("\ c.. OSIC.. \\.Q. -

A simple realization of the harmonic Oscillator is a mass, attached to the end of a simple spring as shown in fig.5.1

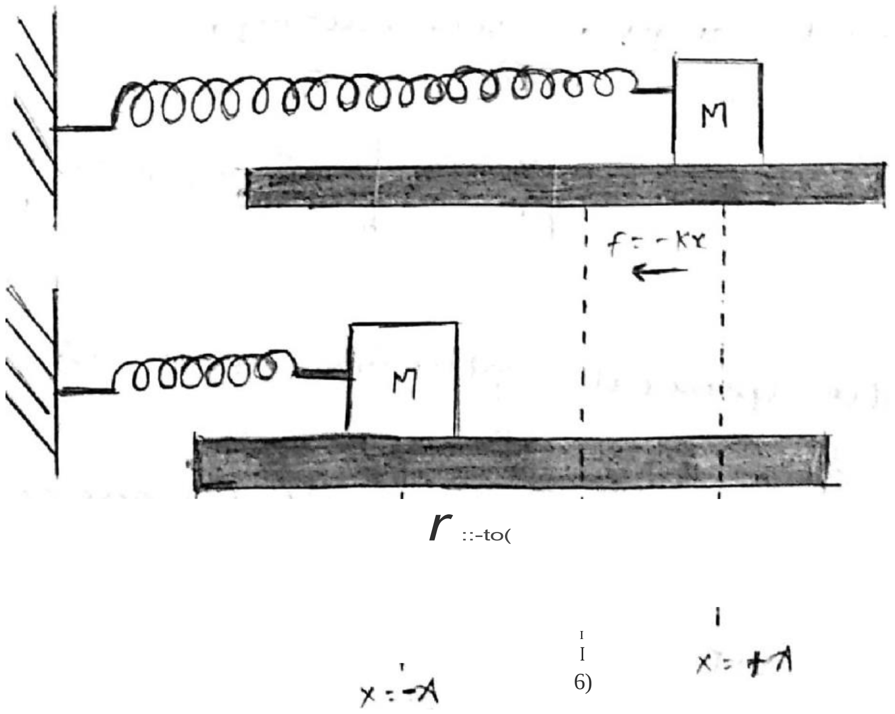

Fig. 5.1 Harmonic oscillator

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The equation of motion for the simple harmonic
[o]scillator is given by

$$\frac{x}{{}^2} \quad + \qquad\qquad \smile$$

W the position of ! t"¼J. o'

[fu]nction of time, t. The constant n

().7., 't\A..L Q..c:::, ""=:: ·, -tvJ. t\,u <;\; CJl.

C..O'C"J:, c:,._n , 'tt.u. \o.:,. ..QJ-. ,n- q S\c:s:'('

given displacement u equ

[po]sition.

Eq. ② can be writ

$$\frac{x}{t^2} \quad \omega\, x \; - \qquad \smile$$

Where the oscillation occur

constant angular

$$\omega = \sqrt{\frac{k}{m}} \qquad \text{since, } \omega^2 = \frac{k}{m}$$

<rt.u- .9_t_n..1. . oJ -2oluhon Ll

$$x(t) = A \qquad\qquad \smile$$

lAJ'tti.A .«–'(''..Q_ n ? c:t\t ('C)O a(') \J.–3 0..

..Q Ln \.J,Jl.. '(ci_oJ \: – (")C)\.OS'\<:A · l<_n \J.J

S. \...f"n? V\.a.::nman.\C. f'r'l c:, o''' a.t"cl tw c...ol'"<"t"{.a.,; ,(:)<;'"\ c--i

g'd <\- .cun ¼,:"\ \.).JC'\ 0:1'. a("\ c. OA \\o. .....\- •

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®

Energy in the Harmonic Oscillator:

0-),_ Q'C'I \..\ £.. CT.\- C<;tc._'-\ \ 0.. +OT

L9. \-t,u. g <n Lb \(_U--...Q_ Q. ..-\ 0-.nd \::-lA.JZ.

QJ-. C:. \:"- OJ.......... Q__ c..a. 'a':\

$$\frac{1}{2}mv^2 + \frac{1}{2}mx^2$$

UJ,-,Jj0l. V \-\Jv- "'-J.o_\.. '- CS\- c\.JU.- \'flc,Y-.JZ-.

When it is 0-.\- 0.. d...\$-\-Q-.. C,,.9- ..;\— equilibrium position.

The study of quantum mechanical harmonic motion begins with the specification of the Schrodinger equation.

c.\. o1 potential energy is given by

f'\{ = $\frac{}{2}Kx^2$

o.'C"d. Sl.O \....X).P- Q_oS"\ '1,:-Q. cl-c,'-.J-'.:) 1tvJ.. Schrodinger equation as

[-' 9-m d: d.rl + L Kf'){_ \\'l""-') E. '\-) (.f1<.)

.Q.("\\-\O..\

•\J,..\..n.Q. e

t""\.o.:5'

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Since harmonic motion has a characteristic angular frequency, it makes sense to measure energy in terms of ω.

Hence, the allowed energies are

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega \quad \text{for } n = 0, 1, 2, 3, \ldots$$

where the ground state is usually designated with the quantum number, n=0

Therefore, we have

$$E_0 = \left(\frac{1}{2}\right)\hbar\omega$$

$$E_1 = \left(\frac{3}{2}\right)\hbar\omega$$

$$E_2 = \left(\frac{5}{2}\right)\hbar\omega$$

$$E_3 = \left(\frac{7}{2}\right)\hbar\omega \quad \text{and so on.}$$

It is clear that the difference between successive energy eigen values has a constant value given by,

$$\Delta E = E_{n+1} - E_n = \hbar\omega$$

The potential energy function and first few energy levels for the harmonic oscillator as shown in Fig. 5.2

⑤

tw. quantum number number,

increases, the energy of the C)S,c_\\O-.,\- and

therefore the amplitude of C.l\.\.O..

increases.

A packet of energy ħω is needed to make the quantum harmonic oscillator to

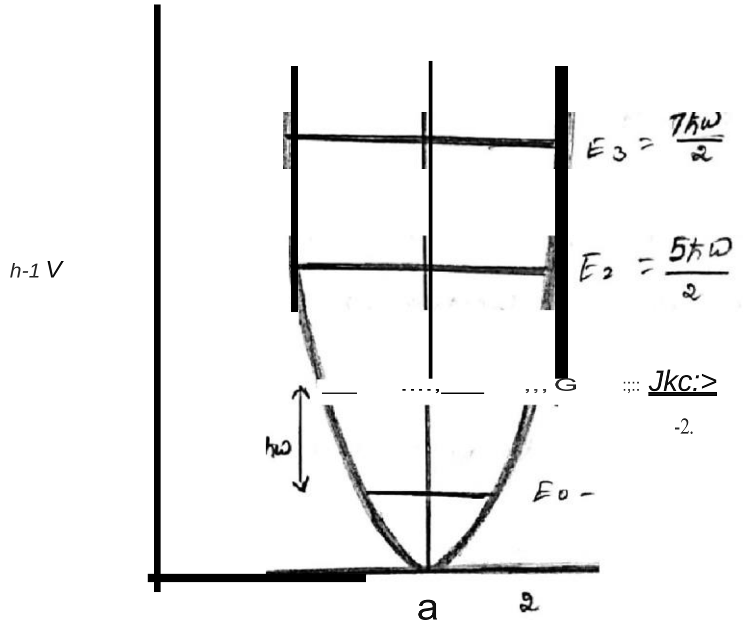
l.'.'.)'J..Q. ('('c::,("(", o.. lo Q'''' ½. 9."c-c...<\.Q ta \A'' c:,

. Q,.oj''\.Ui g_ a...-\;: o. -

Here, the ground-State energy, $E_0 = (\frac{1}{2})\hbar\omega$

0 U.. \"('.o_c,__11;: fua.n c_l Q..w "-l"Q\. G\- ,

\.A_iQ_ Vs a consequence of the uncertainty principle. This means that the oscillator is always oscillating.



$E_3 = \frac{7\hbar\omega}{2}$
$E_2 = \frac{5\hbar\omega}{2}$
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E_0 -
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2

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E = 0 is not allowed by 'O t-\ .o_ I$..S2-(",

ncertainly principle .

ignificance :

) It serves as a prototyp... 0 u-> *Hu* ma. *fiurr.a.*fica

reatment of phenomena the ...

acoustics , AC circuits , molecular and crystal

ibrations , electromagnetic fields and optical

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°'J°u<::>.."-tu..C'C\ ½.. a" Q CS!. \\a.. ·

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5.2. BARRIER PENETRATION AND QUANTUM TUNNELLING:

According to quantum mechanics, a particle such as electron can penetrate a barrier into a region forbidden by classical mechanics.

This phenomenon is known as barrier penetration and can happen only when the particle exhibits wave nature.

Tunnelling is a quantum phenomenon where particles with less energy than that of a potential barrier can still cross the energy barrier, by penetrating through it.

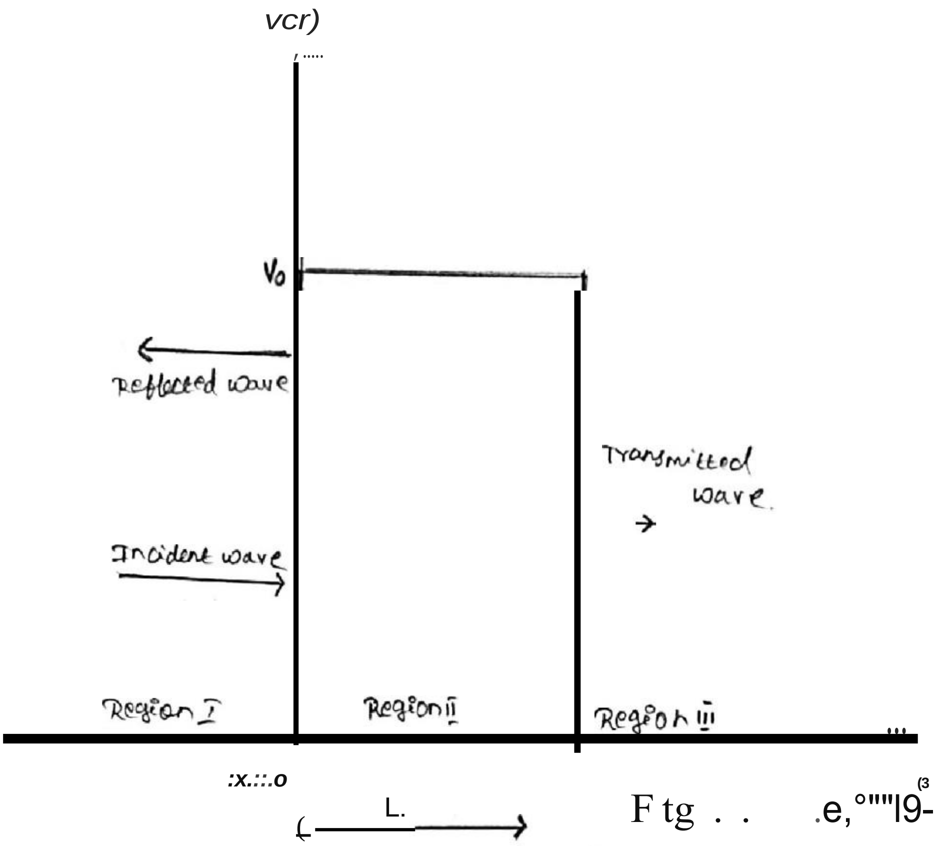
Explanation:

Let us consider a particle of mass m travelling to the right alage the x-axis. The particle encounters a narrow potential barrier whose height Vo is greater than E and whose thickness is L.

- classically all the particles.

(i) Will be reflected back (at $x=0$) if $E < V_0$ and

(ii) Will be transmitted to $x > L$ if $E > V_0$



F tg . . .e,°""l9->- p

But, quantum mechanics .!:)d,,c_tz 0.. '(""\t::,C\- zero probability for finding <t\.N-- Q_\..J1_ the other side of the barrier even when $E<V_0$.

This can happen as the approaching particle has a sinusoidal wave function as shown Fig. 5.4.

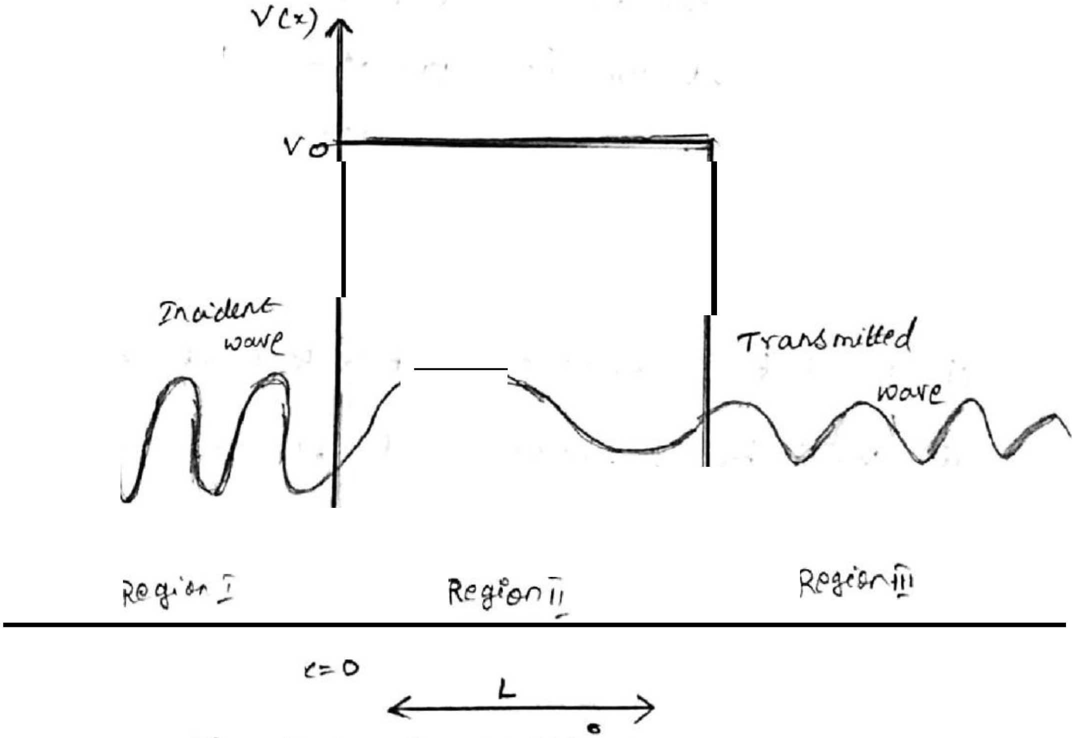


Fig. 5.4. Tunnelling

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Within the barrier the wave is decaying and before it dies away to zero, there is again a sinusoidal wave function, at $x = L$. But it is a sine wave of greatly reduced amplitude.

Since, $[\psi]^2$ is non-zero beyond the barrie it is evident that there is a non-zero probability that the particle penetrates the barrier. This process is called tunnelling through the barrier or barrier penetration. Thus, tunnelling is a result of the wave properties of material particles.

Tunnelling probability:

The tunnelling probability can be described with a transmission co-efficient, T and a reflection co-efficient R.

Since an incident particle must either reflect or tunnel through, we have $T+R=1$

Transmission coefficient:

The probability that the particle gets through the barrier is called transmission coefficient (T).

$$T = \frac{\text{Probability density of transmitted wave}}{\text{probability density of incident wave}}$$

The transmission coefficient is given by

$$T = e^{-2\alpha L}$$

Where,

$$\alpha = \sqrt{\frac{2m(V_0 - E)}{h^2}}$$

The transmission probability increases with decrease in height and width of the barrier.

5.3. TUNNELLING MICROSCOPE :

An electron microscope that works by quantum tunnelling phenomenon and creates atomic scale imaging of surfaces is known as tunnelling microscope.

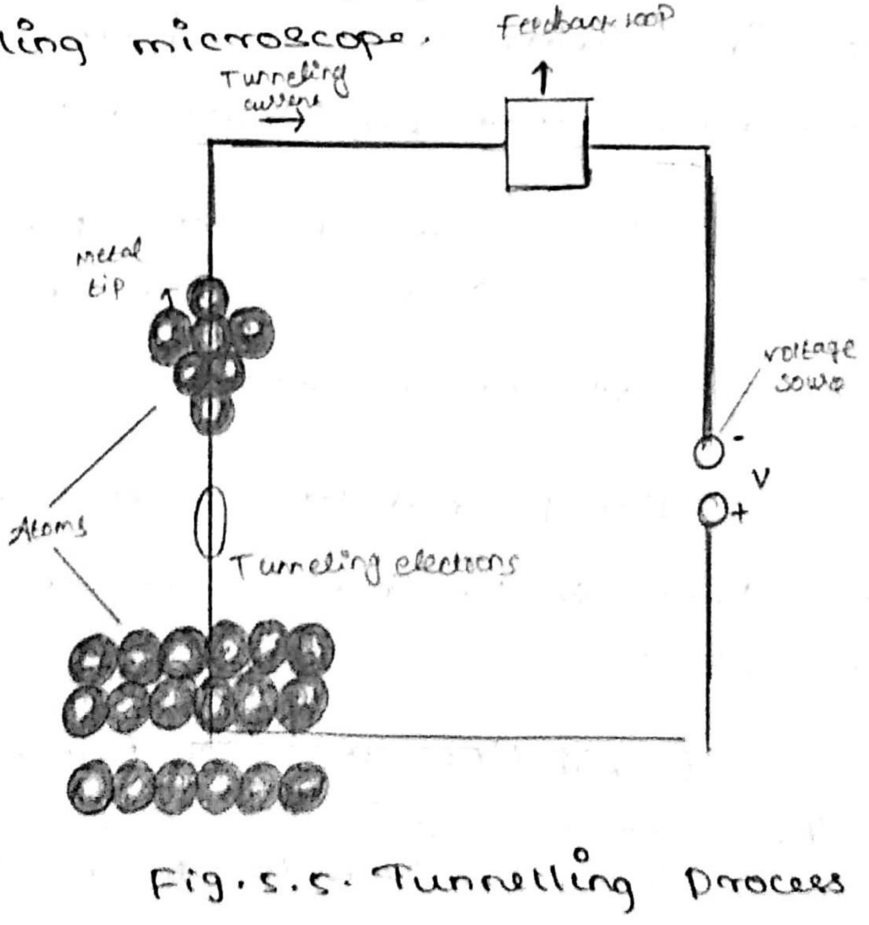

Fig. 5.5. Tunnelling Process

For the electrons in the sample and in the metal tip, it is forbidden to stop in the gap between sample and tip. However, this gap is so small that the electrons are able to tunnel and flow through the gap.

Principle:

When a voltage is applied between a conducting tip and a surface close to it, electron can tunnel through the vaccum between the atoms of the tip and the surface. The tunnelling currents that results depends upon the distance between probe tip and sample surface.

Construction

The basic components are,

- piezoelectric tube
- Tunnelling current amplifier
- Distance control unit & scanning unit
- Data processing and display.

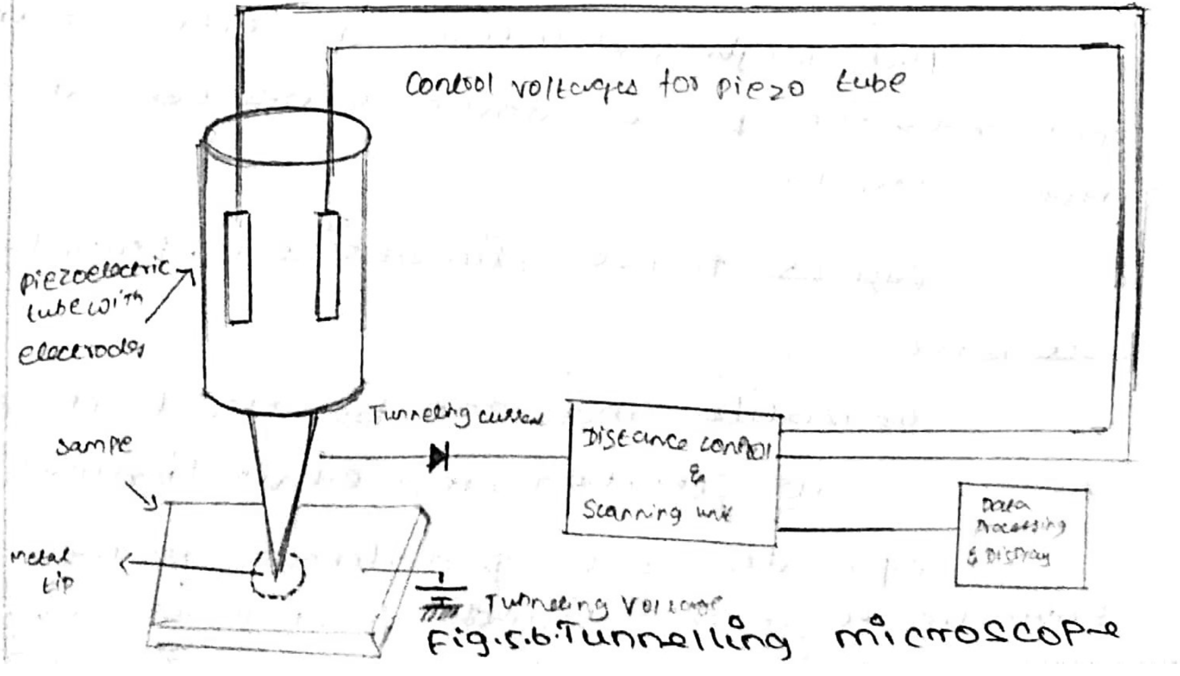
Control voltages for piezo tube
piezoelectric tube with electrodes
Tunneling current
Sampe
Distance control & Scanning unit
Data Processing & Display
metal tip
Tunneling Voltage

Fig.5.6 Tunnelling microscope

Working

The sharp conducting probe tip attached to a piezoelectric tube is positioned at a distance few angstroms from the sample surface.

A small voltage applied between the probe tip and the surface causes electrons to tunnel from the tip to the sample surface.

As the probe is scanned over the surface, the voltage applied to the piezotube is altered to maintain a constant tip-surface distance.

changes in this voltage registers variations in the tunnelling current.

The changes in the tunnelling current are recorded and then used to generate a map of the sample surface on the display uni

Merits:

The high resolution of STMs enables researchers to examine surfaces at an atomic level.

Gives three dimensional profile of a surface.

Versatile and can be used in ultrahigh vaccum, air, water and other liquids and gase

operates in temperatures as low as zero Kelvin up to a few hundred degrees celsius.

Demerits :

Require <V..o_c:-Y S.t-o..olsl , e...t9-o.. g 4 o ("\0X conducting Q C\. :

Difficult to use effectively

'T\.AJ.- Q tStc. o \. 0(-_o..O{ '--9,..9...cl OA.9- QC'R"""-'2.. ('Nl..t sophisticated as well as very expensive.

Applications :

The tunnelling microscope is widely used Ln\:) 'De ttA 0u, olLt"t°r"0" a...\ o_("')d <:\-U-. '(")ciO-'('Y\.9n 't-a.,\ C("'.-..Q.,J:._a_c,.:h<:: !::o Ob"""-U"> o,:tt,c-n. "- - .s c_cS..'- ,;n o..c:.,..sU\ rns>-"=:W .."?.u.J"\c,.c:...s LJ_A,LO(diagnostic tool in the fields t cu ;wG-cf to..-& physics, electrochemistry, chemistry, nano machining etc., b 0Lo9'-) J o<o9o.ncC •

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Used in research s:2 au.- dt..n semiconductors and microelectronics.

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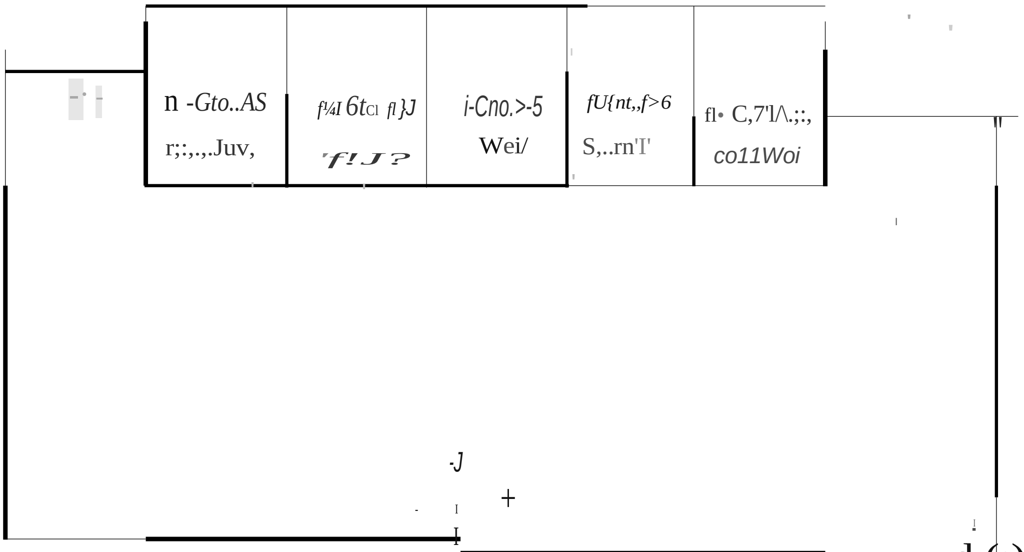
Principle :

<tu._(°""I "'.o\.\ ⁰ r-C\ .Q...\.9- o ('V. t"\A \..,\..q"'- o..
finite - height potential well that occurs only when electron energies match an energy level in the well.

construction :

W .Q. \..).. Q. c--c..QJ. D"a.._0it- o\Jba.
s shown below



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IJ..)o -t\.A n

q\JJ. N""...Q..9⁰'t..a \'lg., 0-.": ,t\1'--1- ..Q.0-.. t"< Q. "o\.J.. D

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c_u..,v, ri-\ D..C-

Working :

According to quantum mechanics, electrons can tunnel from outside into the well through barrier under suitable conditions.

The Energy band diagram of the resonant diode is shown o \.S', fig.5.8

Without any voltage bias, the electron energy level o -LO well is higher than the incident electron energy (E).

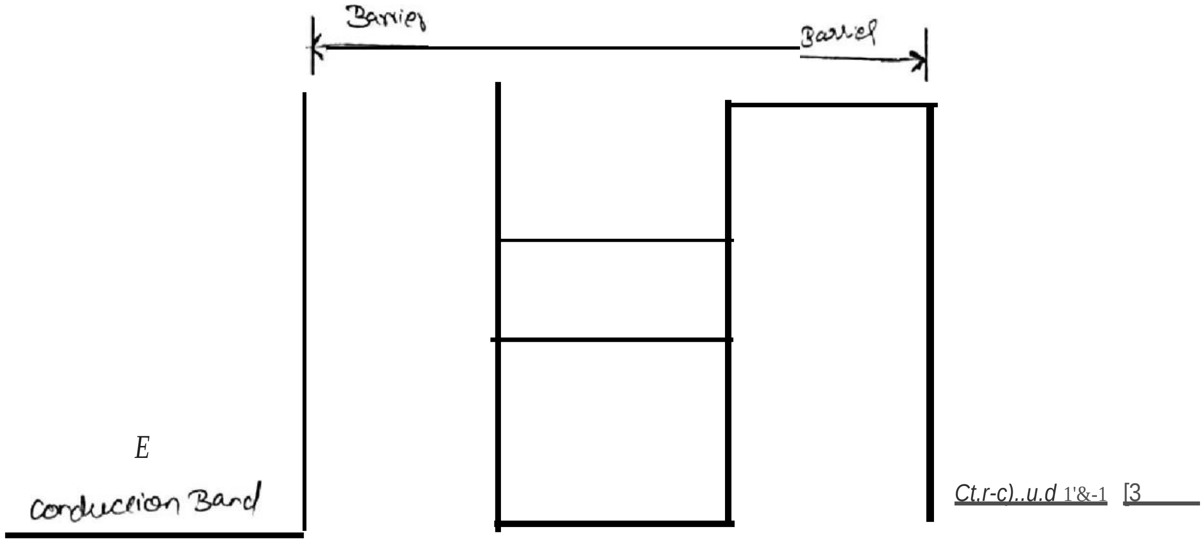


Fig.5.8.Energy B,.o..s, \C\ ..Q_ C"O.'<'. d. 00.S,

c._UJ \'t"v\ "' "J O\<\- -Q ")

So, no Q..t.!] c._ ("") ("') c.a \J.. a<"\

bo...0.d. c_a.._ locU- , e, U,.)..O_\\ o..nd.. u,

ri0 C.....\.., (")\::-

On increasing the bias voltage, the incident electron energy level, E on the left becomes higher and matches an energy level in the potential well. Now some electrons can tunnel into the well and the current from left to right increases due to tunnelling.

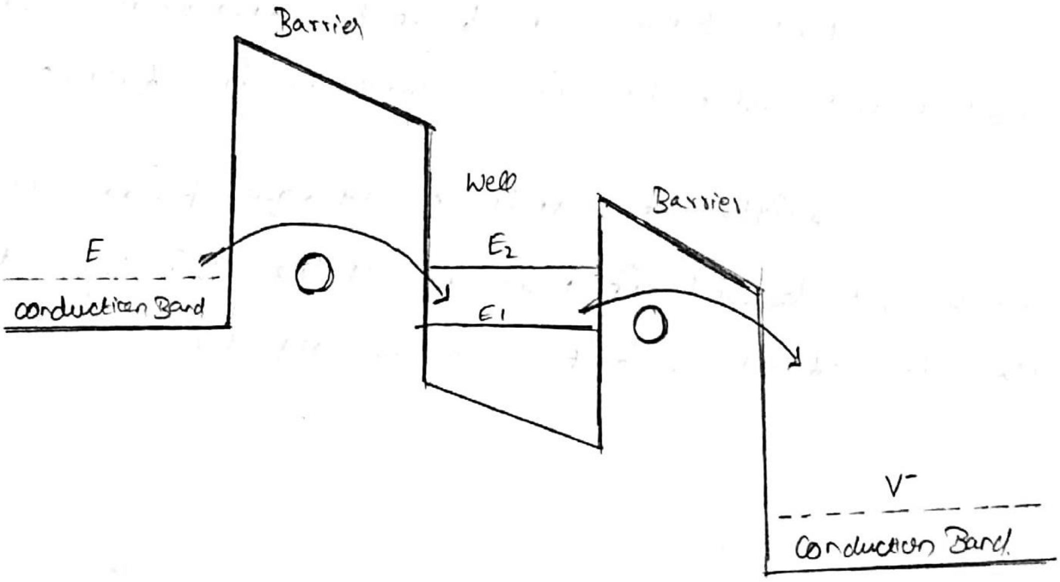


Fig.5.9. Energy Band Diagram of resonant diode (with applied voltage bias)

Thus, when the electric field increases to the point where the energy level of the electrons in the emitter coincides with the quasi-bound state of energy level to the well, the current reaches a maximum. This type of tunnelling is known as resonant tunnelling.

I-V characteristics of resonant diode:

The current - voltage (V) characteristic for resonant diode is shown in Fig.5.10.

As voltage increase, E also increases and hence the tunnelling current increases and reaches a peak point.

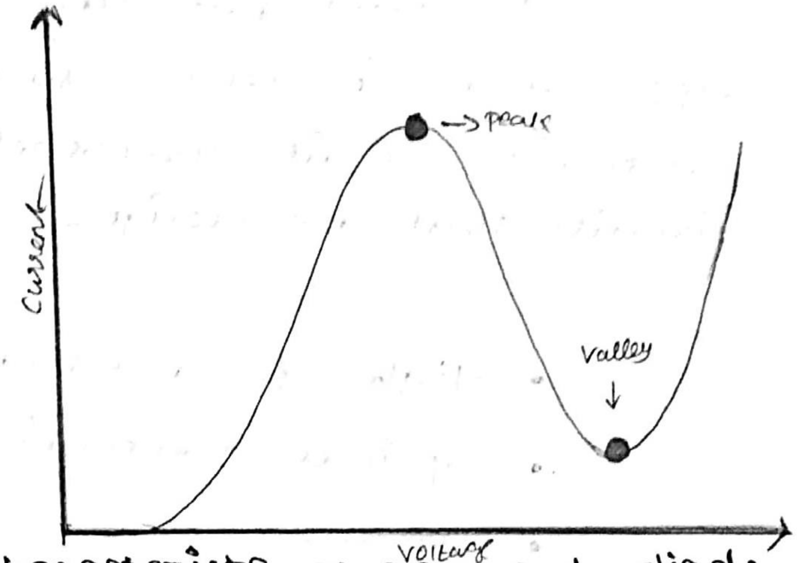


Fig.5.10. I-V-characteristic of resonant diode

Further increases in voltage alters the energy value E and hence the transmission is low. This results in decrease in current which reaches a minimum value point called valley. The decrease of current with an increase in voltage is known as negative resistance.

As the applied voltage continues to increase, current begins to rise again because of substantial thermionic emission.

Let us consider a particle with total energy $E < V_0$ confined within a well bouncing back and forth between the turning points at $x = -L$ and $x = L$.

The potential well is divided into three regions (1, 2, 3) with associated wave function as follows.

$$\psi = \begin{cases} \psi_1, \text{ if } x < -L & \text{(the region outside the box)} \\ \psi_2, \text{ if } -L < x < L & \text{(the region inside the box)} \\ \psi_3, \text{ if } x > L & \text{(the region outside the box)} \end{cases}$$

The time independent Schroedinger's equation can be written as

$$\frac{-\hbar^2}{2m} \cdot \frac{d^2\psi}{dx^2} + V\psi = E\psi \quad \rightarrow ①$$

Inside the box

For this region, inside the box $V(x) = 0$ and equ ① reduces to

$$-\frac{\hbar^2}{2m} \frac{d^2\psi_2}{dx^2} = E\psi_2 \quad \rightarrow ②$$

Letting

$$K = \frac{\sqrt{2mE}}{\hbar}$$

eqn ② becomes

$$\frac{d^2\psi_2}{dx^2} = -K^2\psi_2$$

The general solution for the above differential equation is

$$\psi_2 = A\sin(Kx) + B\cos(Kx) \rightarrow ③$$

Here, A & B are constants and K can be any real number

Hence, $\boxed{E = \frac{K^2\hbar^2}{2m}}$

Outside the box

In the regions, $x < -L$ and $x > L$, we have, $V = V_0$ and now the Schrodinger equation is

$$-\frac{\hbar^2}{2m}\cdot\frac{d^2\psi}{dx^2} + V_0\psi = E\psi$$

We rewrite this as

$$\frac{d^2\psi}{dx^2} - \left[\frac{2m(V_0-E)}{\hbar^2}\right]\psi = 0 \rightarrow ④$$

Let us assume that E is less than V_0, so the particle is "trapped" in the well. There might be only one such bound state or more.

We define a constant G by

$$G^2 = \frac{2m(V_0 - E)}{\hbar^2}$$

and rewrite the Schrodinger equation as

$$\frac{d^2\psi}{dx^2} - G^2\psi = 0$$

This equation has the general solution

$$\psi_{1,3} = Ce^{Gx} + De^{-Gx}$$

In the region, $x < -L$, x is always negative, so, D must be zero. Similarly in region, $x > L$, where x is always positive, C must be zero.

Hence, $\psi_1 = Ce^{Gx}$

$\psi_3 = De^{-Gx}$

Conclusion:

In a finite potential well, the wave function extends into these classically forbidden region where the total energy is less than the potential energy. The situation is possible and is consistent with the uncertainty principle.

However, in both these regions the wave function decreases exponentially with distance from the well.

If the particle manages to acquire energy, $E > V_0$, then it will escape from the well.

An electron confined within a semiconductor by an electric force has a potential energy that can be modelled as a finite potential well. Similarly a proton confined within the nucleus by the nuclear force has a potential energy that can be modelled as a finite potential well. Hence any situation in which a particle is confined can be modelled as a finite potential well.

Characteristics of finite potential well.

The number of bound state energies is finite.

The number of bound states increases with the width and depth of the well.

Tunnelling into the barrier (wall) is possible.

Higher energy states are less tightly bound than lower ones.

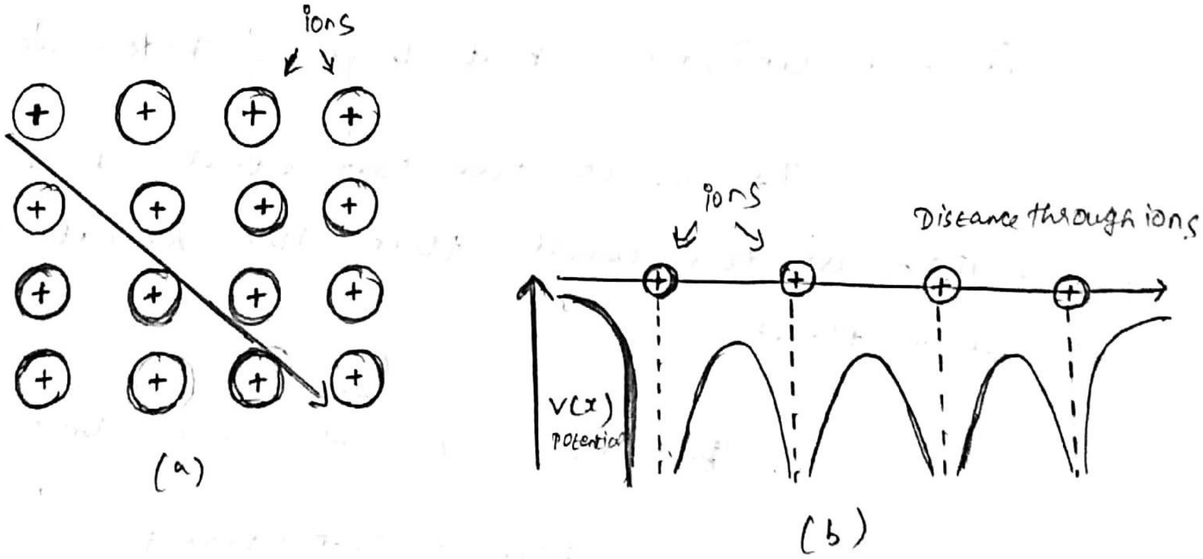
A particle provided with enough energy can escape the well (unbound state).

5.6. BLOCH THEOREM

One of the characteristic features of many solids is the regular arrangement of their atom forming a crystal. The potential energy of electrons in such a crystal is the result of the positively charged ion producing a columbic attraction.

In a crystal, electrons move in a potential $V(x)$ which is produced by regularly-spaced ion cores as shown in fig. 5.12 (a)

The potential of the electron at the site of positive ions is zero and is maximum in between the sites of two positive ions as shown in fig. 5.12 (b)



(a)

(b)

Fig. 5.12. Electrons in a periodic potential

Let a one-dimensional lattice (i.e.) only an array of ionic cores along x-axis is considered.

Since the potential energy of any particle bound in a field of attraction is negative, and since the conduction electron is bound to the solid, its potential energy V is negative.

Further, as it approaches the site of an ionic core, $V \to -\infty$

Since this occurs symmetrically on either side of the core, it is referred to as potential well.

The width of the potential well 'b' is not uniform, but has a tapering shape.

If V_0 is the potential at a given depth of the well, then the variation is such that,

$$b \to 0, \text{ as } V \to \infty \text{ and hence,}$$

$$bV_0 = \text{constant}$$

Now, since the lattice is a repetitive structure of the ion arrangement in a crystal, the type of variation of also repeats itself.

If "a" is the inter-ionic distance, then, as we move in x-direction, the value of V will be same at all points which are seperated by a distance equal to "a".

$$\text{ie, } V(x) = V(x+a)$$

where, x is distance of the electron from the core.

Such a potential is said to be a periodic potential.

The Bloch theorem states that, for a particle moving in a periodic potential, the eigen functions for a conduction electron are of the form,

$$\psi(x) = U(x)\cos Kx$$

$$\text{where, } U(x) = U(x+a)$$

The function U(x) has the same periodicity as the potential energy of the electron, and is called the modulating function.

Significance:

A large number of materials are well described by regular atomic spacing and a periodic potential for a crystal lattice which is like a string of finite wells.

The presence of periodic potential in a crystal leads to energy bands, which are essentially energy intervals between which energy levels are nearly continuous.

5.7. THE KRONIG PENNEY MODEL:

The study of essential behaviour of electrons by approximating the potential inside a crystal to the shape of rectangular steps is called Kronig - Penney model of potentials.

The Kronig - penney model describes the one dimensional representation of electron potential in a periodic lattice (Fig. 5.13)

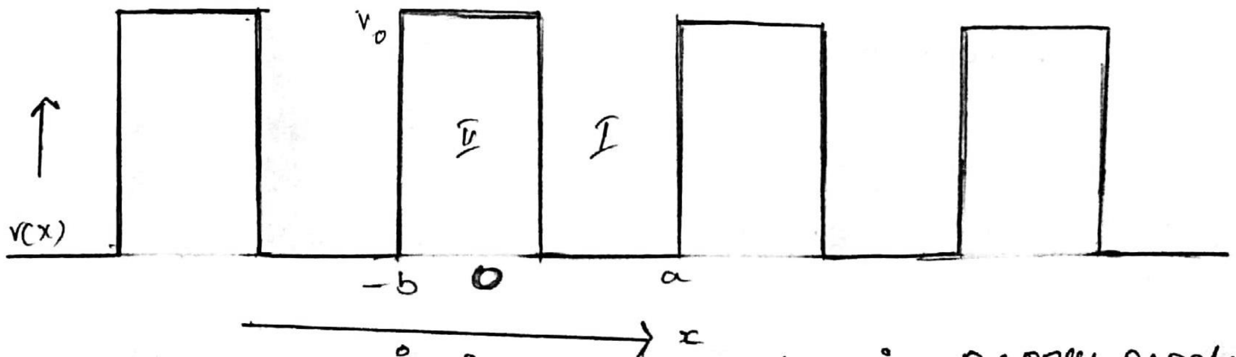


Fig: 5.13 periodic potential - Kronig - penney model

This model consist of an infinite row of rectangular potential wells separated by barrier of width "b". Each well has a width "a" and a depth V_0.

It is assumed that when an electron is near the positive ion site, potential energy is taken as zero. whereas, outside the well, that is, in between two positive ions, potential energy is assumed to be V_0.

Hence, we have

$$V(x) = V_0 \quad \text{for} \quad -b < x < 0$$

and

$$V(x) = 0 \quad \text{for} \quad 0 < x < a$$

The period of the potential is, $(a+b)$

The possible states that the electron can occupy are determined by the schrodinger equation,

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

The schrodinger equation for the two regions can be written as

Region I

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}(E)\psi = 0 \quad , \; 0 < x < a \rightarrow ①$$

$$\left[\text{since} \; , \; V = 0\right]$$

Region II $\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}(E - V_0)\psi = 0$, $-b < x < 0$ → (2)

[since, V = 0]

We rewrite the above equation as

$$\frac{d^2\psi}{dx^2} + \alpha^2\psi = 0 \text{ for } 0 < x < a \rightarrow (3)$$

and

$$\frac{d^2\psi}{dx^2} - \beta^2\psi = 0 \text{ for } -b < x < 0 \rightarrow (4)$$

where,

$$\alpha^2 = \frac{2mE}{\hbar^2}$$

$$\beta^2 = \frac{2m}{\hbar^2}(V_0 - E)$$

Bloch has given the solution for Schrodinger equation as

$$\psi(x) = U(x)\cos kx$$

where, $U(x) = U(x+a)$

Solving the above equation (3) & (4) by applying boundary conditions, we get

$$\frac{8\pi^2 m V_0}{2h^2 a} b \cdot \sin(\alpha a) + \cos(\alpha a) = \cos(ka) \rightarrow (5)$$

$$\frac{P}{\alpha a}\sin(\alpha a) + \cos(\alpha a) = \cos(ka) \rightarrow (6)$$

where, $\frac{\pi^2 m V_0 \alpha b}{h^2}$ potential area strength

$\alpha a \sin(\alpha a) + \cos(\alpha a)$

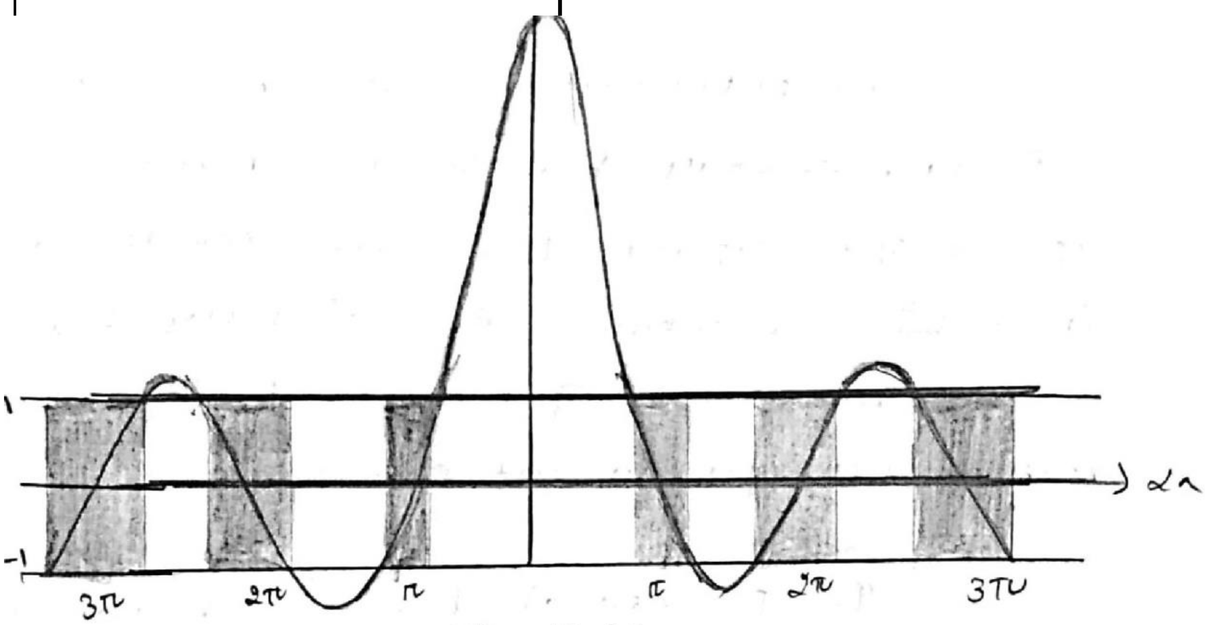


Fig. 5.14

By varrying CCJ(_> o., wo.. C\.u... '- '''- nechanical nature could be plotted g \AQU.....'<"l fig. 5.14.

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Results from Kronig - Penney Model:

The Kronig - penney model demonstrates that a simple one dimensional periodic potential yeilds energy bands as well as energy band gaps.

If potential barrier between wells is strong, energy bands are narrowed and speed far apart. This corresponds to crystals in which electrons are tightly band to ion cores, and wave functions donot overlap much with adjacent cores.

If potential barrier between wells is weak, energy bands are wide and spaced close together.

The energy spectrum of electrons consist of an infinite number of allowed energy bands seperated by intervals in which, there are no allowed energy levels. These are known as forbidden regions.

When "α" increases, the width of the allowed energy band also increases and forbidden energy regions become narrow

5.8. ORIGIN OF ENERGY BAND (OR) BAND THEORY OF SOLIDS:

An easy way to consider how energy bands arise in a solid is to look at what happens to the energy levels of isolated atoms as they are brought closer and closer together to form the solid.

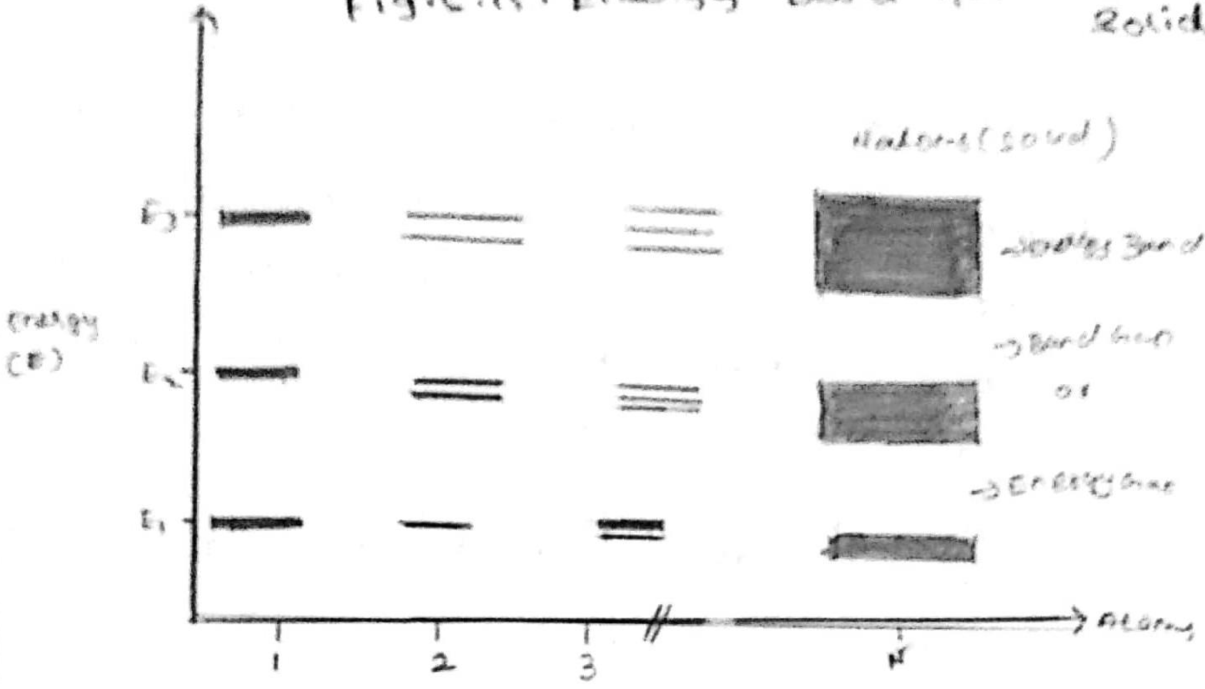
In an isolated atom, the electrons are tightly bound and have discrete, sharp energy levels.

As the atoms are brought closer together, each of the energy level for each atom changes because of the influence of the other atom.

Hence, when two identical atoms are brought closer, the outermost orbit of these atoms overlap and interacts with the wave functions of the electrons of the different atoms, then the energy levels corresponding to those wave functions split into two.

Thus, an energy level split into two levels of slightly different energies for the two-atom system and three levels of slightly different energies for the three-atom system and so on as shown in fig.5.15

Fig.6.15. Energy band formation in a Solid



When a p large number of atoms (of order 10^{23} or more) are brought together to form a solid, the number of orbitals becomes exceedingly large, and the difference in energy between them becomes very small. So the levels may be considered to form continuous bands of energy rather than the discrete energy levels of the atoms in isolation.

Within an energy band, energy level are so numerous and are so close to each other that they form almost continuous band.

However, some intervals of energy contain no orbitals i.e., the forbidden energy levels, no matter how many atoms are aggregated, forming band gaps.

KEY POINTS:

Energy Band

A set of closely packed energy level is called as energy band.

Width of a band

The overall range of energies from the lowest to the highest level for a band is called the width of a band

Valence band

A band which is occupied by the valence electrons is called as valence band. The valence band may be partially or completely filled up depending on the nature of the material.

Conduction band

The lowest unfilled energy band is called as conduction band. This band may be empty or partially filled. In conduction band the electrons can move freely.

Forbidden gap or band gap:

The energy gap between valence band and conduction band is called forbidden energy gap or forbidden gap or band gap.